STUDY OF MODEL MATCHING TECHNIQUES FOR THE DETERMINATION OF PARAMETERS IN HUMAN PILOT MODELS

REPORT ON TASK 2

LINEAR, TIME - VARIANT MODELS

8426-6003-RU-000 20 November 1963

Prepared under Contract NAS1-2582
For NASA/LANGLEY RESEARCH CENTER

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ABSTRACT

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This report describes the results of a study of continuous model matching techniques as applied to the determination of parameters of time-varying human pilot models. This work constitutes Task 2 of a study of model matching techniques being conducted under NASA Contract NAS1-2582.

The intital phase of the study is aimed at an improvement of convergence time of the continuous model matching technique developed under Task 1. The dependence of convergence time on the choice and composition of the criterion function, on parameter adjustment gain and on filtering in the adjustment loop are studied. Results obtained in this phase are used in matching artificially perturbed parameters of a second order system by adjusting the parameters of a second order model. The original system is made time-variant by perturbing its parameters sinusoidally or stepwise. The final portion of the study applies the model matching technique to the determination of parameters in a mathematical model of a human pilot engaged in performing a time-varying control task.

The major conclusion of the study is that continuous on-line parameter tracking techniques can indeed be applied to the determination of parameters in time-varying systems. However, considerable caution is required in interpreting the resulting parameter variations in the mathematical model since these may be influenced not only by the actual variations in the system but also by a number of extraneous effects. These effects are traced in some detail and methods for reducing their influence are suggested.

Author

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1. INTRODUCTION

This report presents the results of Task 2 of a research program concerned with the development of automatic computer techniques for the determination of parameters in mathematical models of human pilots. The major objective of Task 2 is the application of these techniques to the case where the pilot behavior can be assumed to be time-varying, under the influence of time-varying controlled element characteristics.

Based on the results of Task 1 the continuous model matching technique was selected for Task 2. It was clear however that a number of unanswered questions from Task 1 required consideration before the continuous method could be applied to a time varying situation. In particular, it was necessary to optimize the convergence time for the fixed parameter case, since the requirements on rate of convergence become considerably more severe if the parameters are allowed to vary. Consequently, the first phase of the Task 2 study (and the first section of this report) describe the results of a series of tests which were performed to improve the performance of the continuous method in the fixed parameter case. The effect of forcing function initial conditions and adjusting loop gain on the behavior of the parameters was examined. The contribution of a rate term in the error criterion was also studied carefully and considerable improvement in performance was observed as an error rate term was added. During these investigations a number of interesting effects were observed, including large angular excursions of the gradient vector which are reflected in the descent trajectory in the parameter space, and cross coupling effects among the parameters. Considerable progress has been made in obtaining an analytical explanation of these phenomena which are discussed in detail in Section 4 of this report.

Following the first phase the improved continuous method was applied to the determination of parameters in a mathematical model of a system made intentionally time-varying by sinusoidal or stepwise perturbation of one of its coefficients. The final phase of the study made use of the technique in the analysis of human tracking performance under conditions where gain and one time constant of the controlled element were intentionally varied.

The notation throughout this report emphasizes the use of differential equations for characterizing system dynamics. Transfer functions are avoided almost completely. This approach is necessary since transfer functions represent the frequency domain behavior of time-invariant systems. As time variations are introduced, considerable caution is required in applying frequency domain techniques while the differential equation clearly and unambiguously describes the system behavior. While it is true that such phrases as "time-varying poles and zeros" do appear in the literature, the interpretation of these phrases is open to considerable question and consequently they are avoided entirely in this report.

2. EXPERIMENTAL PROCEDURE

The experimental procedure utilized in Task 2 was essentially identical to that of Task 1, and consequently the descriptions given in Reference 1 are applicable. As described above, Task 2 was divided into three phases; the experimental procedure used in these phases is summarized as follows:

Phase 1

In this phase the system to be identified had fixed parameters. The experimental procedure was identical to that used in Task 1. A change in the error criterion was introduced for the purpose of improving the convergence time. The new error criterion is described in connection with the results in Section 3.

Phase II

This phase was concerned with the identification of a time-varying parameter in the original system. A coefficient potentiometer in the analog simulation of the system to be modeled was replaced by a multiplier, and a sinusoidal or square wave was used to perturb the system parameter. The parameter adjustment circuit of the model system tracked the parameter perturbations.

Phase III

In this phase the technique of Phase II was applied to indentification of human pilot model parameters while the pilot performed a single-axis compensatory tracking task using a finger-tip controller as described in Reference 1. The controlled element, or plant, was made time-variant by the following time sequence:

For the first two minutes of a five minute run the plant was described by the differential equatio..

$$\ddot{p} + \dot{p} = 20y$$

or by the transfer function

$$\frac{P}{Y} = \frac{20}{s(s+1)}$$

During the third minute of the run the gain was increased at a constant rate for 10 seconds until it reached a value of 45. This value of gain was held for one minute and 50 seconds. During a subsequent 10 second interval the plant differential equation was changed at uniform rate to

$$\ddot{p} + (1 - 0.01t)\dot{p} = 45y$$

After these ten seconds, the plant transfer function was given by

$$\frac{P}{Y} = \frac{45}{s^2}$$

and these characteristics were maintained for 50 seconds. At the end of four minutes of tracking the plant was again returned to its initial state and remained in this condition for the final 50 seconds of a five-minute tracking run. The same configuration was used in four runs by two operators. As in Task 1 the data were recorded on magnetic tape and subsequently analyzed by the continuous model matching method.

3. RESULTS AND DISCUSSION

As outlined in the previous section the study was divided into three major portions. The first portion of the study was concerned with a determination of maximum allowable adjustment gains, the choice of criterion functions, and the effect of initial conditions and forcing functions on the behavior of the tracking system. As a result it was possible to optimize the techniques which were to be employed in dealing with time-varying parameters in the system to be modeled. As in Task 1, the system equation used for methods development was similar in form to the equation to be employed in modeling the human pilot. The second portion of the study included both sinusoidal and stepwise parameter variations. The final portion of the task was concerned with application of the model matching technique to the determination of parameters in a model of a human pilot performing a time-varying control task.

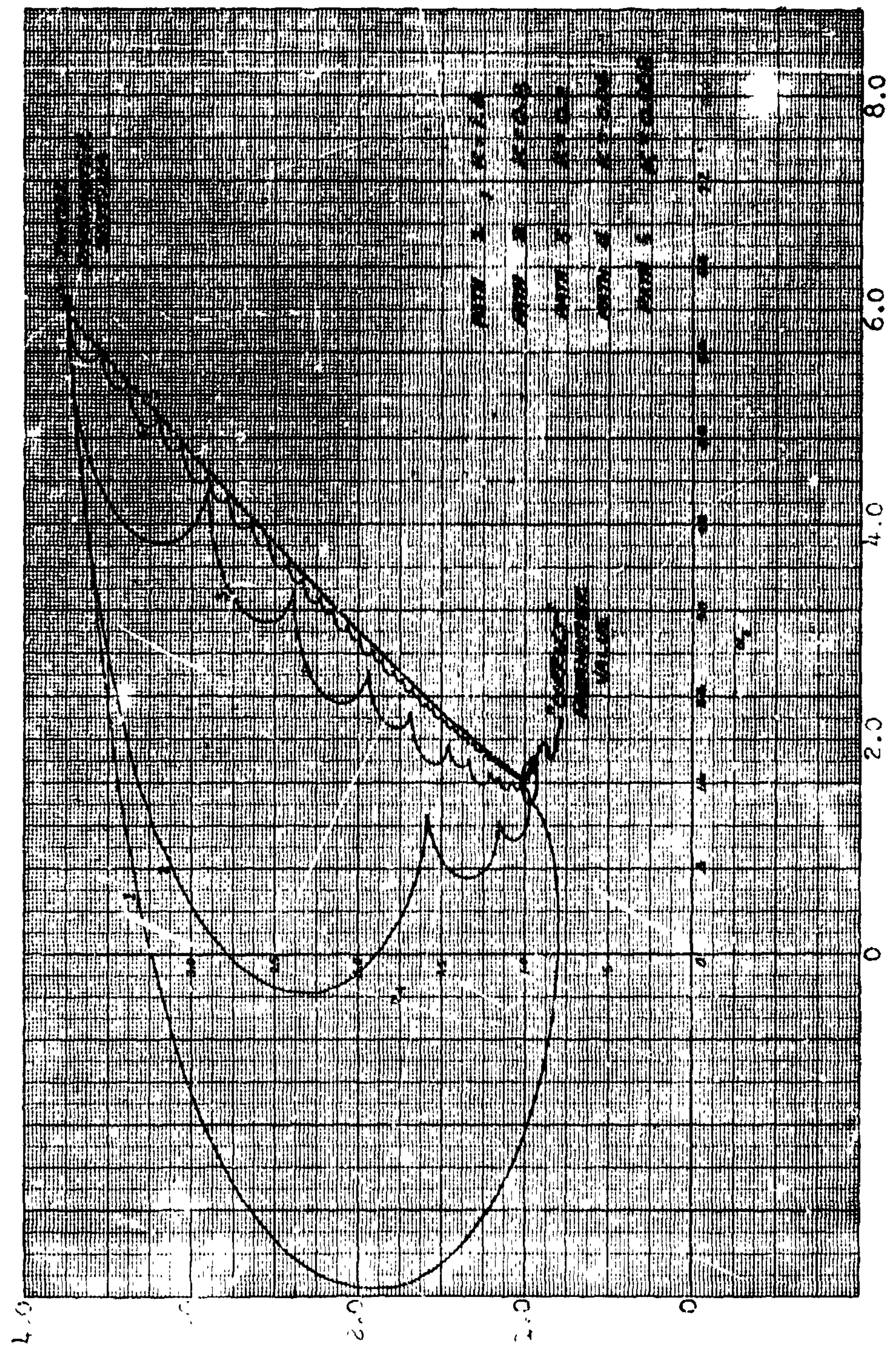
3.1 Improved Model Matching of a System with Fixed Parameters

In order to optimize the convergence time in the case where the system to be identified has fixed parameters, the behavior of the continuous model matching technique was examined for both sinusoidal and random inputs. In both cases the effect of adjustment gain, initial conditions, and criterion functions on the adjustment path was examined. This study was initiated with sinusoidal inputs in order to permit a convenient analysis of the effects of the forcing function and of the adjustment process on the behavior of the parameters. The results of Task 1 appeared to contain random disturbances in the parameter adjustment process. The use of sinusoidal inputs in the present task helped to clarify the nature of this apparent randomness, which proved to be actually a systematic response characteristic inherent in the method used.

3.1.1 The Effect of Adjustment Gain

Consider first the effect of the gain on the behavior of the parameters in the adjustment process, as indicated in Figure 1 for sinusoidal input and Figure 2 for a random input. Both of these figures show descent trajectories in the α_3 vs. α_4 plane for various values of adjustment gain. The parameters α_1 and α_2 were held fixed. Corresponding time traces showing

Descent Trajectories in α_3, α_4 Plane - Sinusoidal Excitation



α_3

Figure 1.

the behavior of the parameters α_3 and α_4 vs. time for one value of gain are presented in Figures 3 and 4 respectively. An examination of these four diagrams reveals the following interesting performance characteristics:

(1) The adjustment process ceases at instances where the matching error changes sign. This occurs once every 1/2 cycle for a sinusoidal input and at randomly spaced intervals for random inputs. Consequently, the trajectories in the parameter plane exhibit a scalloping or vignetting effect. As expected, the "scallops" exhibit periodic characteristics for sinusoidal input and a random sequence of amplitudes for the random input case.

(2) The adjustment paths in the parameter plane depend strongly on the adjustment gain, but approach a direct un-scalloped descent path as the gain is reduced. This can be observed most clearly in Figure 1 for the case of a sinusoidal input. Paths 1, 2, 3, and 4 progressively approach path 5.

(3) The length of time required to converge to within a specified accuracy of the true parameter values can be measured in Figure 1 by counting the total number of scallops. Each scallop corresponds to one half cycle of the driving frequency, in this case 1.0 radian per second.

(4) One observes that the most direct path obtained by the lowest value of adjustment gain requires the longest time to converge. This path therefore involves the smallest approximation errors in gradient computation due to time-variance of parameters. Clearly the most rapid descent, which deviates widely from the direct path, at times does not approximate a gradient path at all. It can be noted that the deviation of this path from the asymptotic path does not necessarily represent an instability or a faulty behavior of the adjustment system. Thus path number 1 in Figure 1 differs drastically from path number 5, and yet results in the fastest convergence. Furthermore it can be noted that high adjustment gains, such as those of path 1, result in a limit

cycle oscillation about the desired final value.

(5) Consider path 4 in Figure 2 which represents the adjustment of parameters α_3 and α_4 with a random input and a low value of adjustment gain. It can be seen that successive path deviations from a direct descent differ in magnitude, depending on the characteristics of the forcing function. It is important to note that such variations in the trajectory are directly traceable to the forcing function and bear no relation to the actual parameter being tracked, which in this case is fixed. This observation must be kept in mind when attempting to match human tracking data:oscillations in parameter values do not always reflect actual variations in human operator behavior; they may simply be introduced by the random behavior of the forcing functions. This effect can also be observed in Figures 3 and 4 which show the random patterns of adjustment as a function of time.

3.1.2 Effect of Initial Conditions of the Forcing Function

Consider now the effect of the initial phase of the forcing function upon the adjustment process. This effect is illustrated in Figure 5 for a sinusoidal forcing function. This figure shows the effect of initial conditions approximately 180° out of phase. It can be seen that trajectories 10 and 12 which were obtained from approximately equal conditions differ drastically from trajectories 11 and 13 obtained when the input was applied one quarter of a cycle later. The effect of the initial condition can be seen primarily in the initial stages of the adjustment process. After approximately two cycles the trajectories converge. The importance of this effect must be considered if random inputs are used since in that case the phasing of the adjustment process is arbitrary and an exact repetition of trajectories from run to run cannot be expected even when matching fixed parameters.

3.1.3 Effects of Parameter Initial Values

Figure 6 shows the trajectories obtained with a sinusoidal excitation signal for various initial values of parameters α_1 and α_2.

Descent Trajectories in α_3, α_4 Plane - Random Excitation

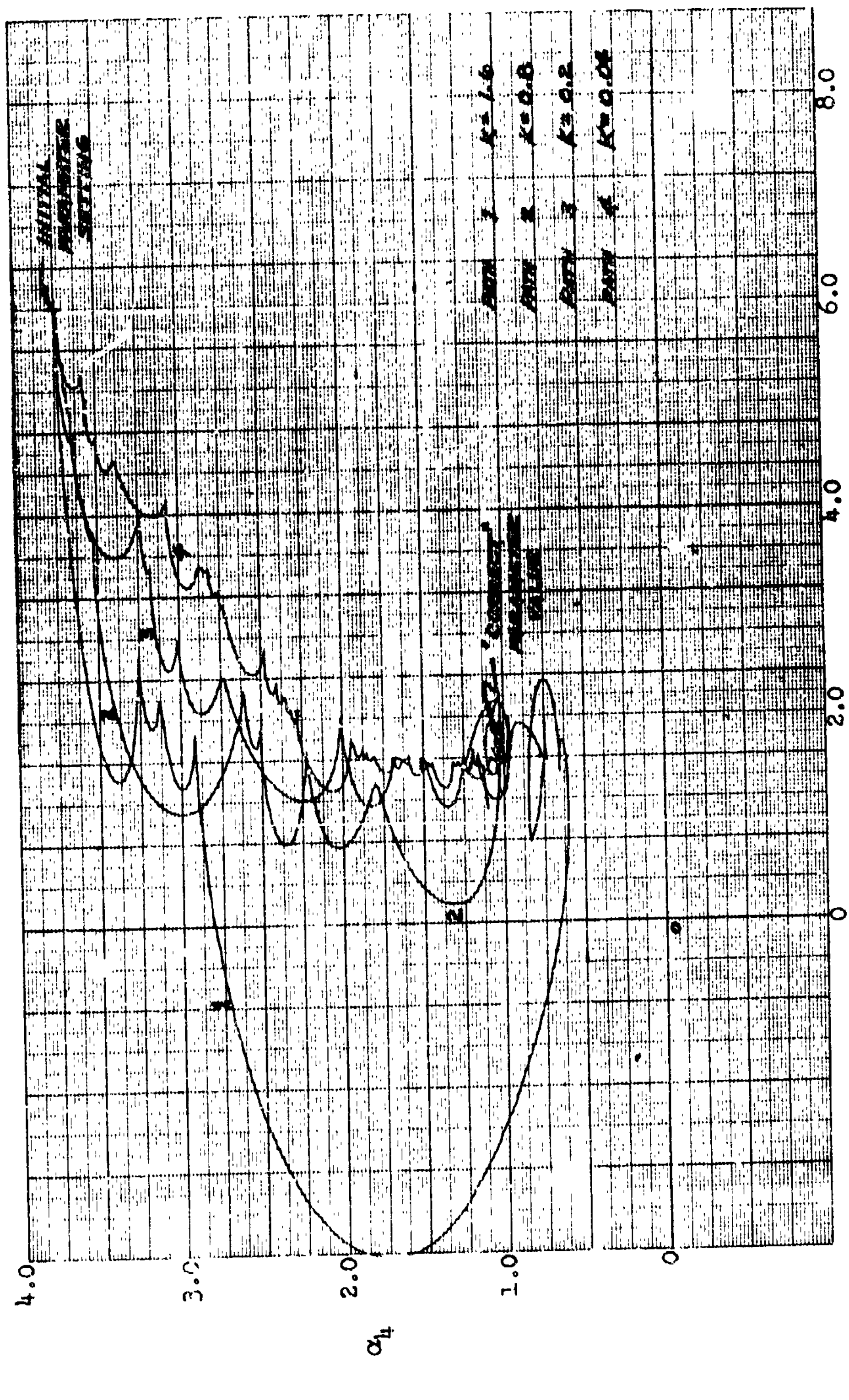


Figure 2.

Time History - Parameter Adjustment of α_3 and α_4
Sinusoidal Excitation

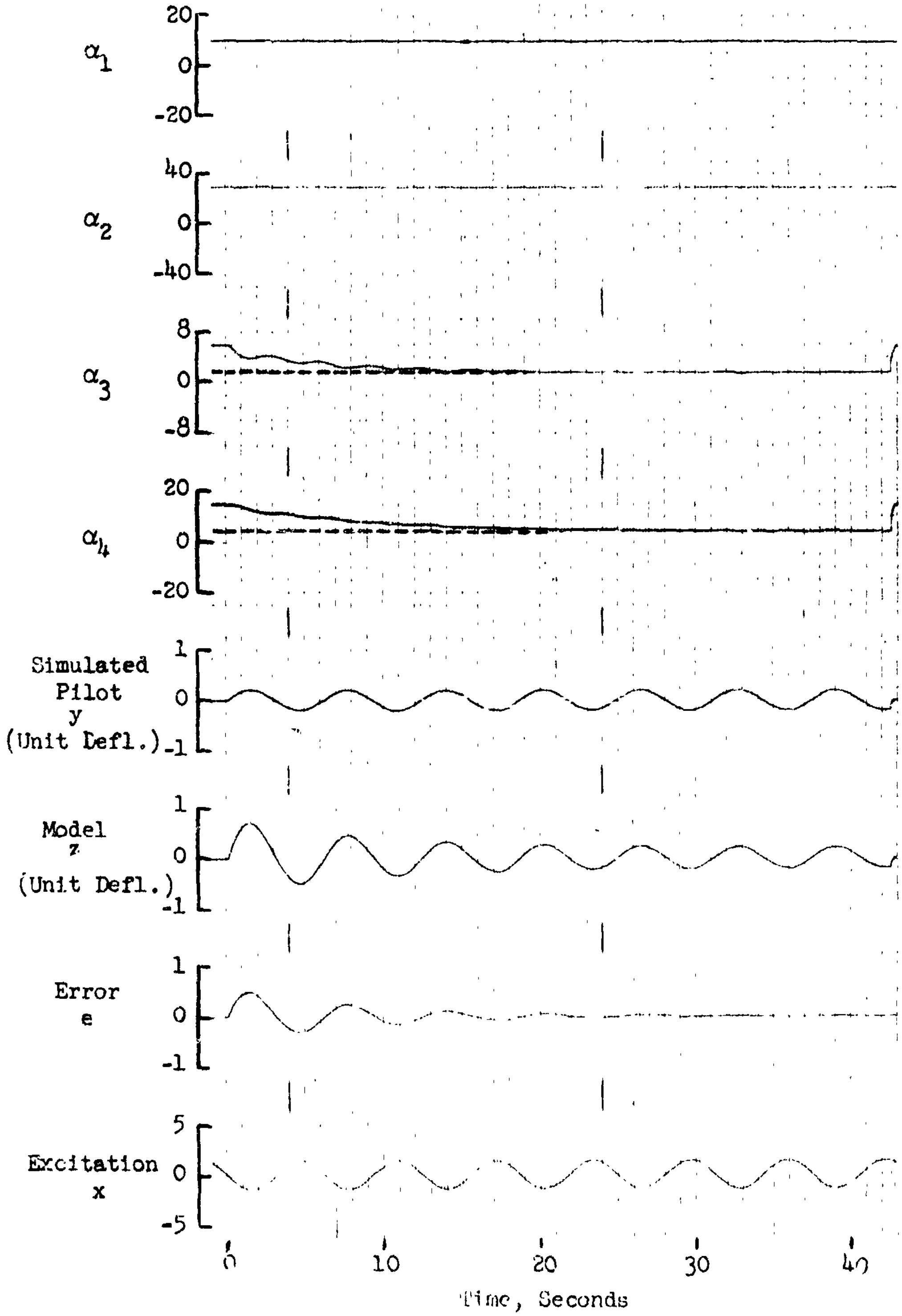


Figure 3.

Time History - Parameter Adjustment of α_3 and α_4
Random Excitation

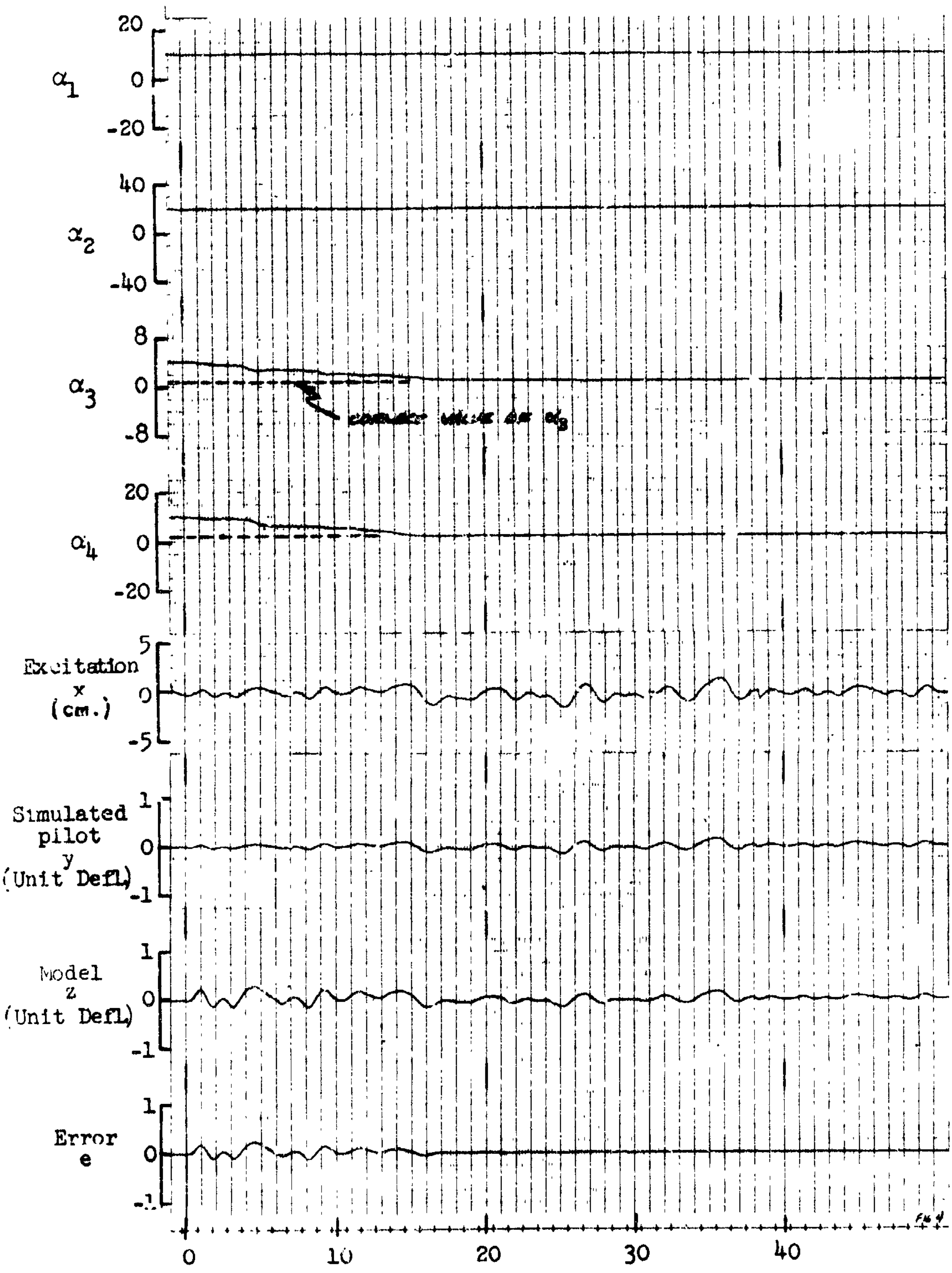


Figure 4.

Effect of Excitation Initial Conditions on Trajectory in α_3, α_4 Plane - Sinusoidal Excitation

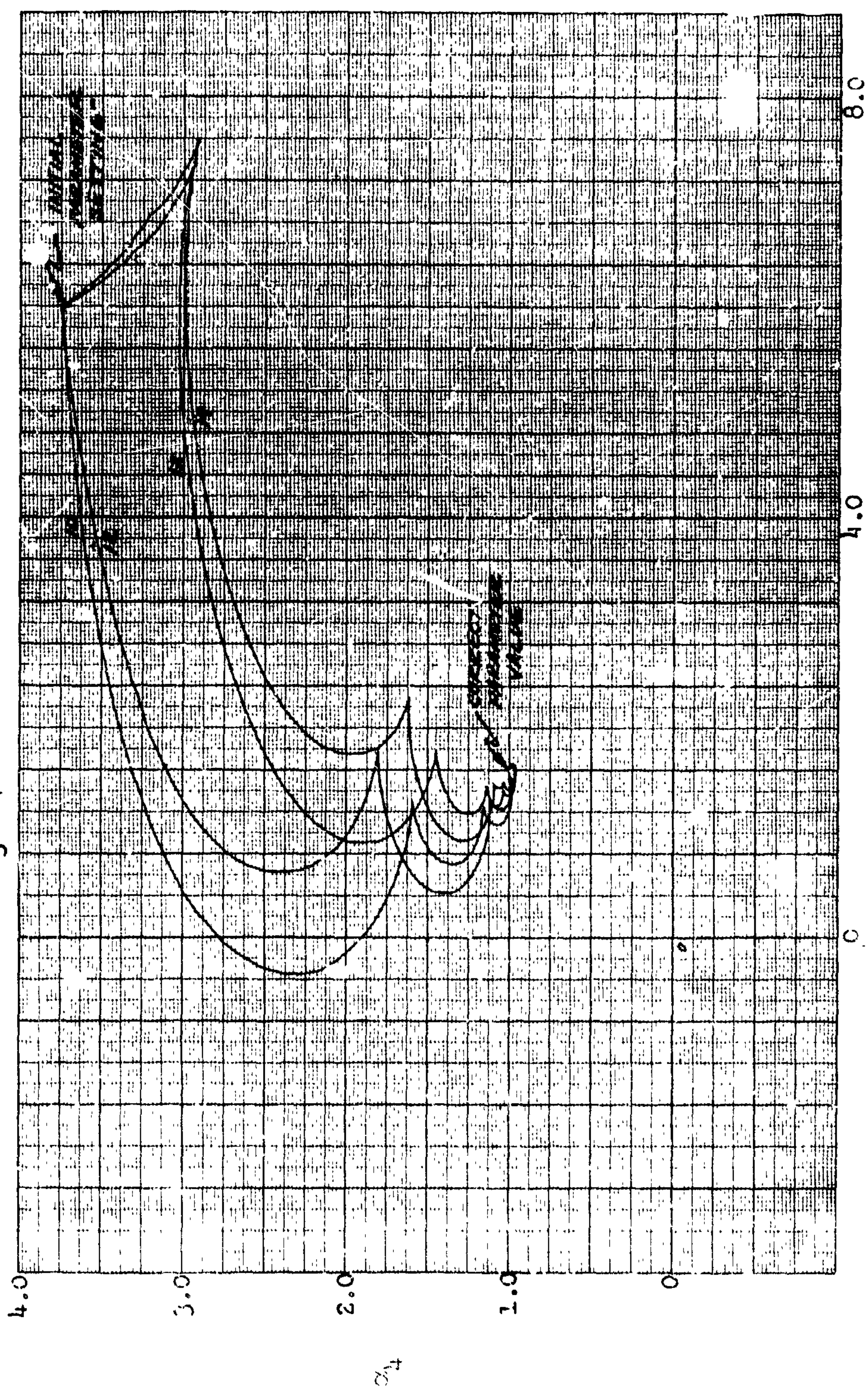


Figure 5.

Effect of Parameter Initial Conditions on Trajectories in α_1, α_2 Plane
Sinusoidal Excitation

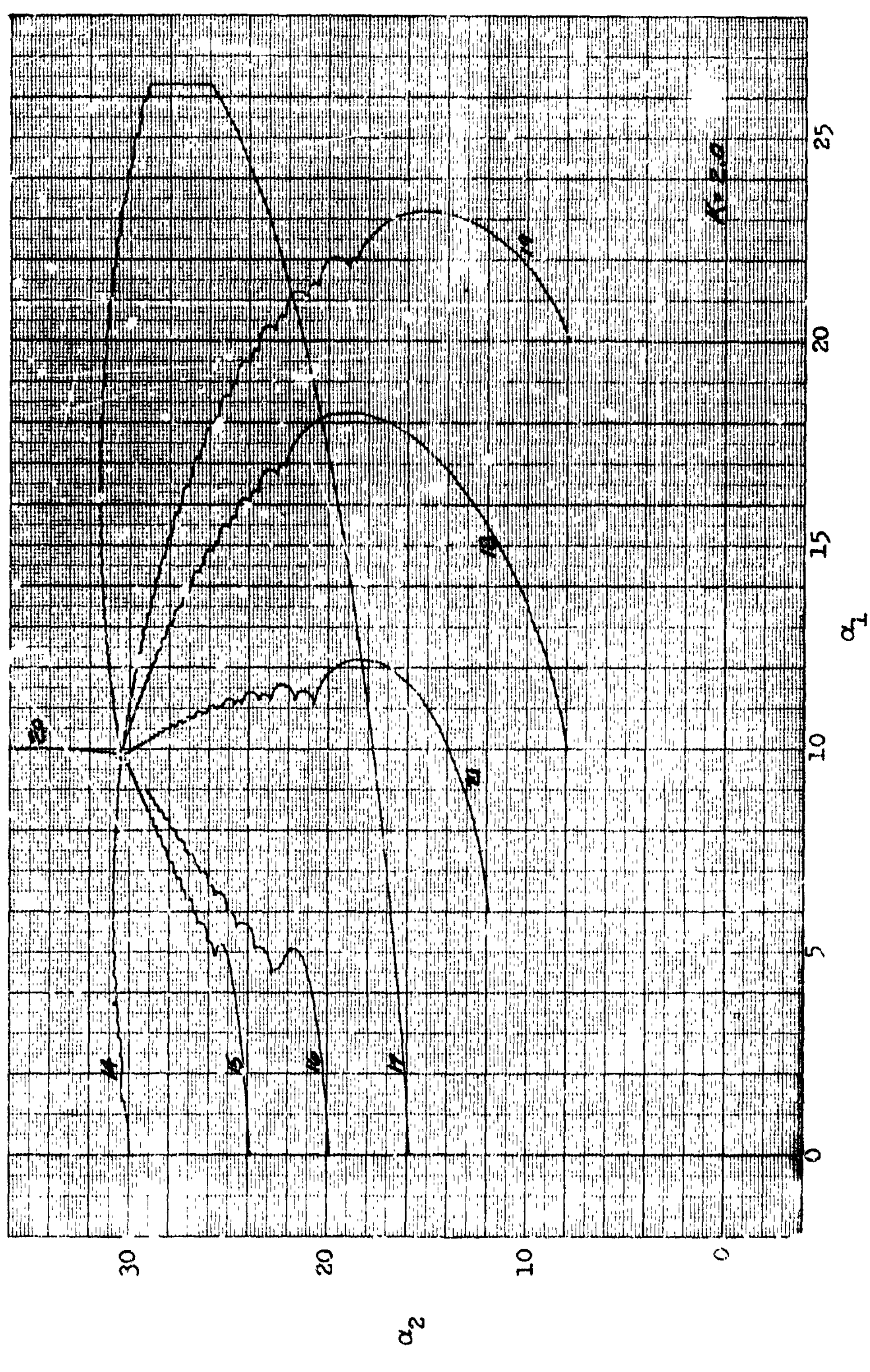


Figure 6.

The typical scalloping effect is again observable. The forcing function initial condition is approximately the same for all the runs shown in this figure. The wide excursions obtained for some initial values of α_1, α_2 again illustrate the sensitivity of the trajectories to the phasing of the forcing function. The descent paths of Figure 6 also give a general indication of the structure of the criterion function in terms of contour lines in this plane. Since each trajectory approaches the true gradient path, contours of the criterion function must intersect the terminal trajectories of Figure 6 approximately at right angles. Hence, the contour lines are approximately of elliptic shape with major axes in α_2-direction.

3.1.4 Effects of Rate Terms in the Criterion Function

The criterion function used during the continuous model matching runs in Task 1 was chosen as the square of the model matching error, i.e.

$$F = \frac{1}{2}\epsilon^2 \tag{1}$$

The work of Margolis (Reference 2) suggests a considerable improvement in convergence time if a term proportional to the rate of change of the matching error is added to the criterion function. Consequently, the criterion function of the form

$$F = \frac{1}{2}(\epsilon + q\dot{\epsilon})^2 \tag{2}$$

was selected where q is a constant. Different values of q were used in the study to find optimum conditions.

In order to obtain rapid convergence (which is desirable for the tracking of time-varying parameters) it is necessary to increase the adjustment loop gain. However, the parameter adjusting loop becomes unstable when gain is sufficiently high. This effect is illustrated in Figures 7, 8, and 9 for q = 0, that is, when no rate term is present in the criterion function. These figures illustrate the behavior of the four parameters under the following conditions:

a. Parameters α_1 and α_2 were set at their "correct" value, i.e., $\alpha_1 = a_1$ and $\alpha_2 = a_2$.

b. Parameters α_3 and α_4 have initial offsets.

c. The input forcing function was a sine wave of frequency 1.0 rad/sec.

d. The following adjustment gains were used in the adjustment loop:

Figure 7	$K = 0.04$	(equal for all parameters)
Figure 8	$K = 2.0$	
Figure 9	$K = 16.0$	

An examination of these three figures reveals the following interesting facts:

(1) When the adjustment gain is very low ($K = 0.4$) as in Figure 7, it can be seen that the convergence of parameters α_3 and α_4 to their correct values is extremely slow. After 50 seconds of running neither parameter has attained its correct value.

(2) When the gain is increased to $K = 2.0$, the parameters approach to within 5 percent of their correct values in 5 seconds. It is interesting to note that with this value of gain parameter α_3 displays a low level oscillation, and that the rapid adjustment is reflected by displacements in parameters α_1 and α_2. Cross coupling effects of this type will be discussed in Section 4.

(3) As the adjustment gain is increased to $K = 16$ instability occurs in parameter α_3, while parameter α_4 shows a very slightly damped oscillation. The cross coupling effect is very strongly evident, being revealed by pronounced oscillations in parameter α_1 and incorrect values in parameter α_2. It can be seen that attempts to increase the gain much beyond the value $K = 2.0$ used in Figure 8 produce instability rather than improved convergence.

Time History - Adjustment of Four Parameters
With Sinusoidal Excitation
q = 0   K = 0.04

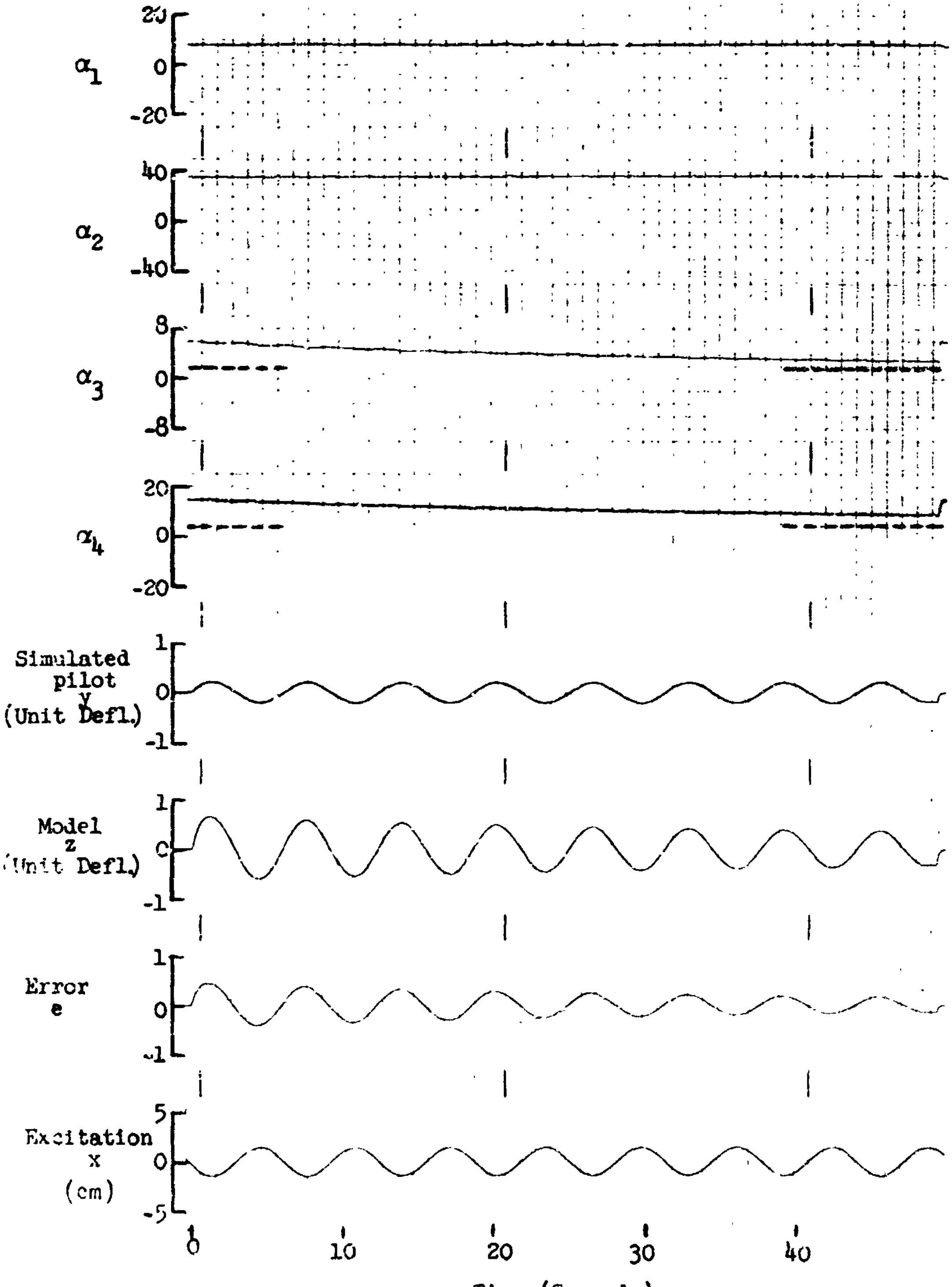


Figure 7.

Time History - Adjustment of Four Parameters
With Sinusoidal Excitation
$q = 0 \quad K = 2.0$

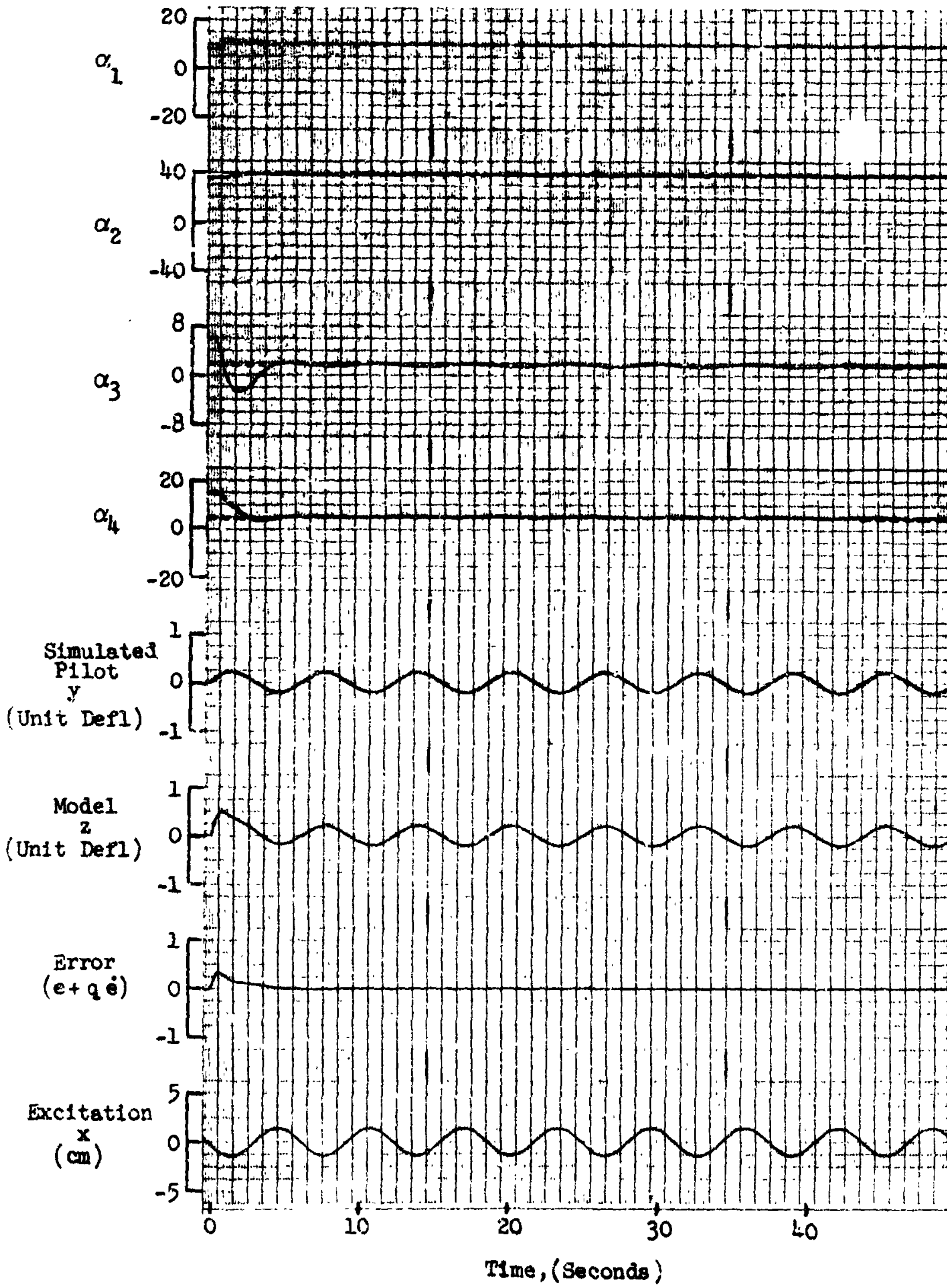


Figure 8.

Time History - Adjustment of Four Parameters
With Sinusoidal Excitation
q = 0   K = 16.0

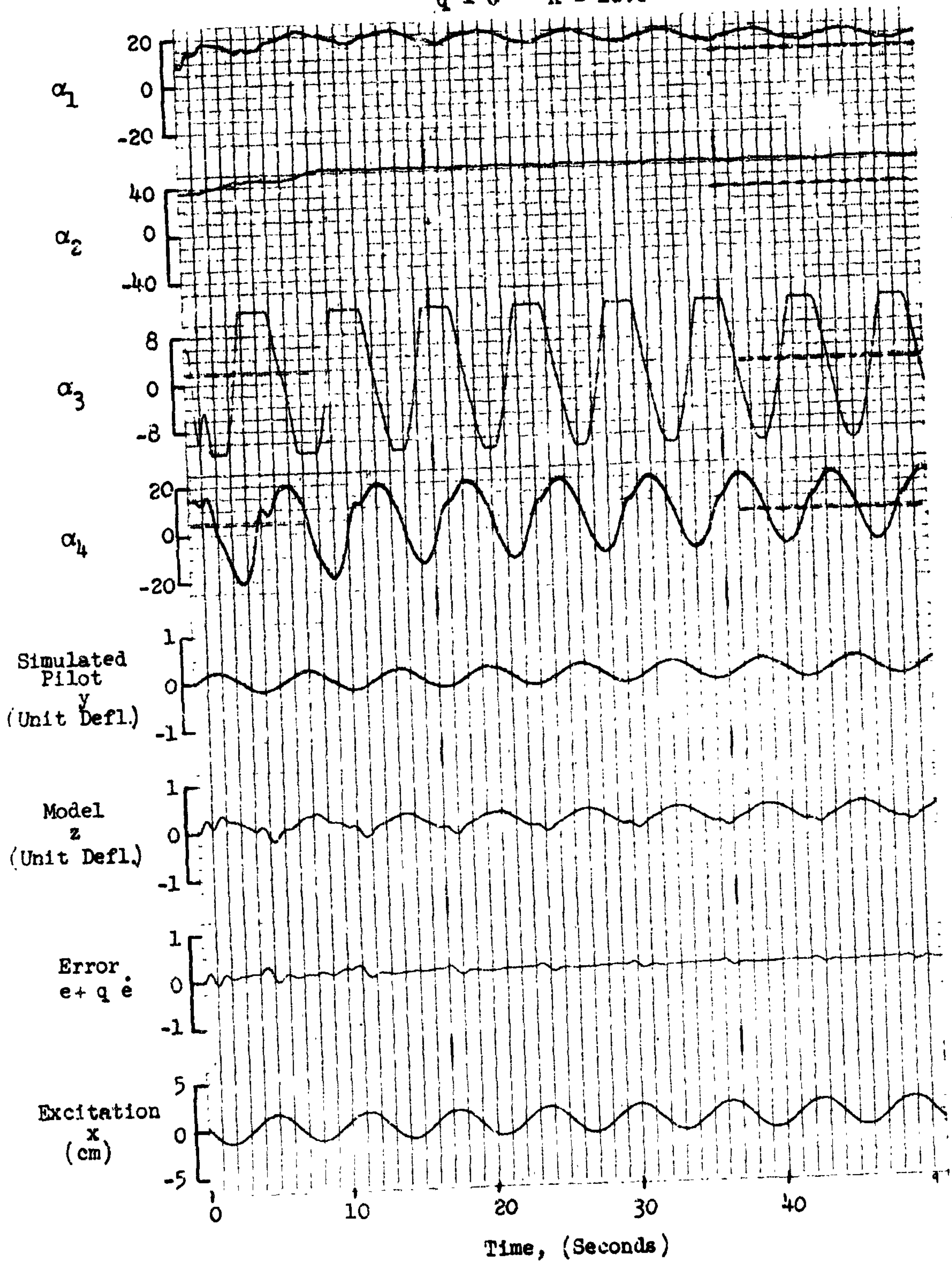


Figure 9.

Differences in stability behavior of the different parameters will also be discussed in Section 4.

(4) It is interesting to note from an examination of Figure 9 that the instability in the parameter adjusting circuits results in only slight disturbances in the matching error. The model output is approximately sinusoidal with small "humps" introduced by limiting from the parameter adjustment circuits. However, the matching error remains extremely small. Comparison of system output and model output does not reveal the unsatisfactory behavior of the parameter adjusting circuits.

Consider now the effect of increasing the contribution of the rate term by adjustment of the rate coefficient q in Equation (2). The effect of setting q equal to zero, 0.5, and 1.0, respectively, is seen in Figures 10, 11, and 12. The adjustment gain in each of these three figures is held at $K = 8.0$, that is, the value of gain is selected sufficiently high to result in oscillatory but not quite unstable behavior of the parameter adjusting circuits in the absence of the rate term. An examination of these three figures shows the dramatic improvement in performance which occurs as q is increased from zero. For $q = 0.5$, most of the oscillation in the parameters disappears and the criterion function is essentially zero throughout the duration of the run. It should be noted with reference to Figures 10 and 11 that, as before, an examination of model output and system output does not reveal the oscillatory behavior of the parameters. As q is increased to 1.0 the oscillation in parameter α_4 disappears entirely while that in α_3 is reduced to less than 5 percent of its maximum value. Convergence of the parameters to within 5 percent of the desired values occurs in approximately 3 seconds. These results show that high values of gain yielding rapid convergence without instability can be tolerated by the parameter adjustment circuits when a criterion function of the form of Equation (2) is used.

Before proceeding to the application of the method to the tracking of time-varying parameters, it was necessary to investigate the effect of the rate term q on parameter adjustment when a random forcing function was used. This effect is illustrated in Figures 13, 14, and 15

Effect of Error Criterion on the Time History of Parameter Adjustment - Sinusoidal Excitation

K = 8 q = 0

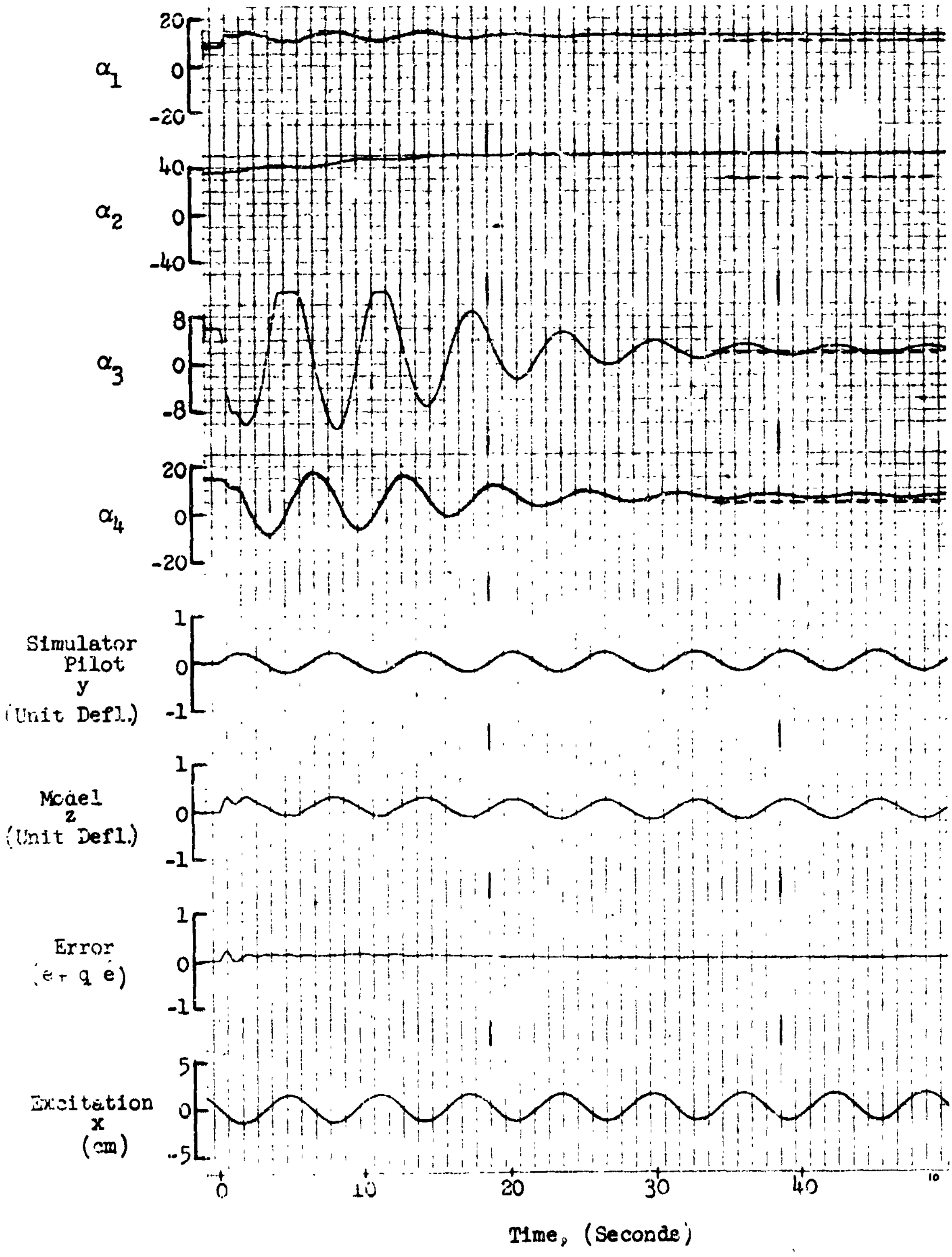


Figure 10.

Effect of Error Criterion on the Time History of Parameter Adjustment - Sinusoidal Excitation
K = 8   q = .5

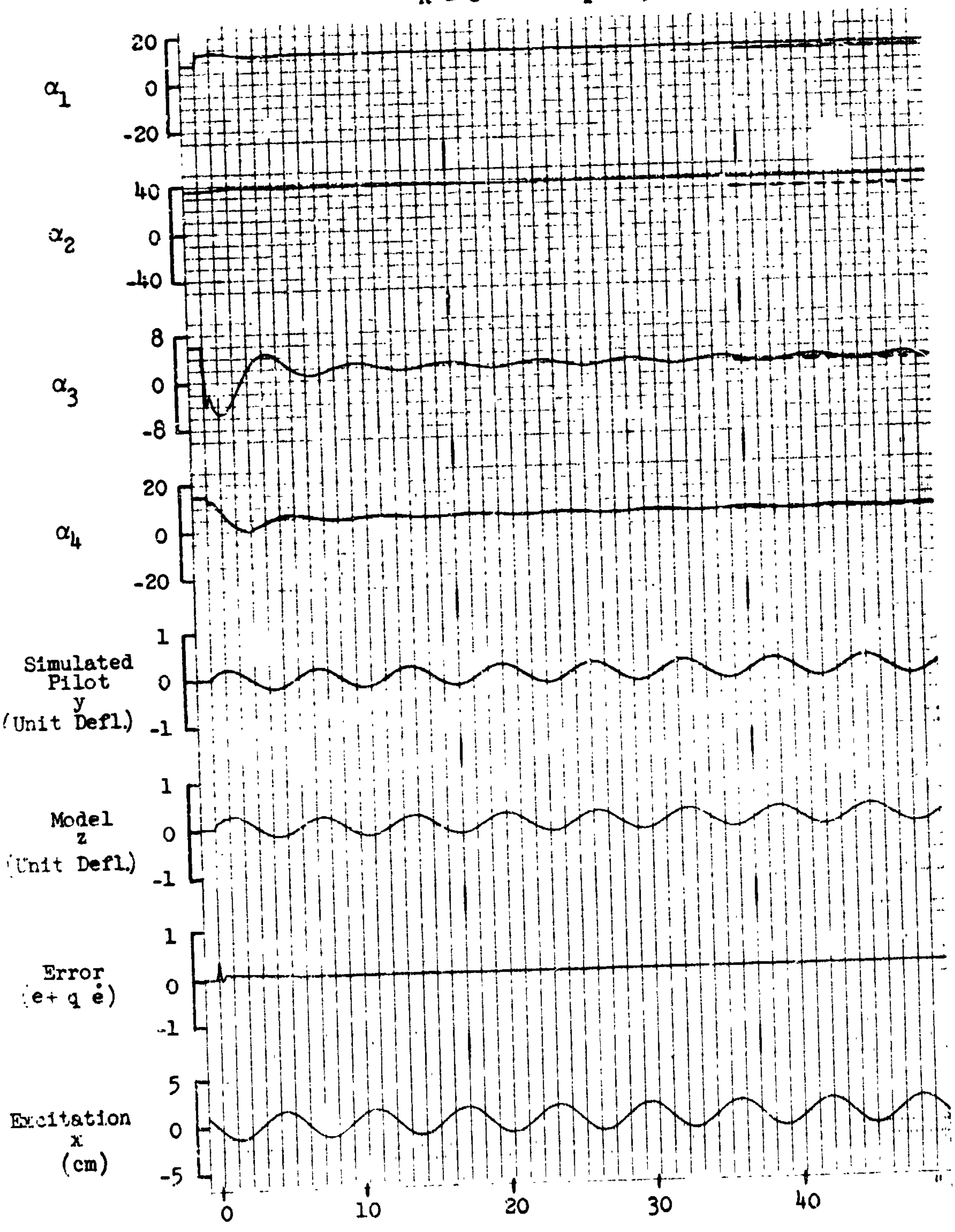
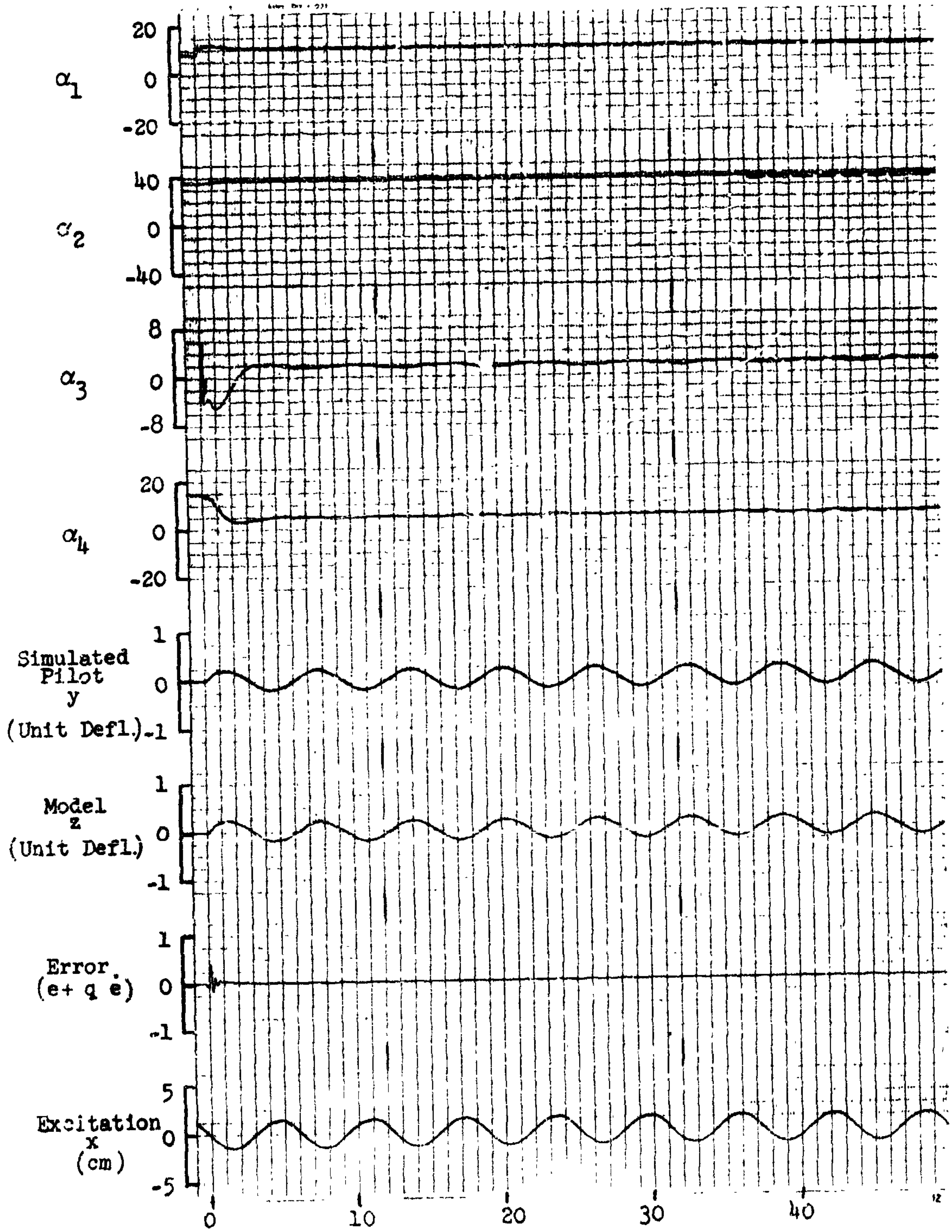


Figure 11.

Effect of Error Criterion on the Time History of Parameter Adjustment - Sinusoidal Excitation

K = 8 q = 1.0



Time, (Seconds)

Figure 12.

where the following conditions apply:

(a) The input forcing function was a random signal obtained by filtering the output of a noise generator, using a third order lag filter with a break frequency of 1.0 rad/sec. (This signal is identical to that used in Task 1 of this study).

(b) The adjustment loop gain was K = 16 for parameters α_3 and α_4. This value corresponds to an adjustment gain which resulted in instability for a sinusoidal input.

Parameters α_1 and α_2 were not allowed to vary in order to minimize the interaction between parameters.

(c) The values of q used in Figures 13, 14, and 15 were 0, 0.5 and 1.0 respectively.

Figures 13, 14, and 15 again show the dramatic improvement in performance which is obtained by the addition of the rate term to the criterion function. Figure 13 shows instability of the parameter adjusting circuits in the absence of the rate term. It is interesting to note, that due to the nature of the input function there are portions of the tracking run when the parameters remain approximately constant and the matching error approaches zero. However, at times when the forcing function makes large excursions the equilibrium is disrupted and the parameters begin to oscillate.

Figure 14 shows the behavior of the system with q = 0.5. Parameters α_3 and α_4 converge to within approximately 5 percent of their correct values within one second and exhibit small random oscillations ($\pm$ 5 percent from the correct value) during the entire run. The improved convergence time, as compared to the 3 second adjustment with the sinusoidal input, is probably due to the presence of higher frequencies within the excitation signal. It can also be noted that the system output and the model output are essentially equal i.e., the matching error is nearly zero. Increasing the rate term to q = 1.0 does not result in further improvement of the parameter adjustment process. The overshoot in the adjustment of parameter α_3 in Figure 15 is probably not caused by the increased value of q, but rather by the effect of an unknown initial condition in the forcing function, as discussed previously.

Effect of Error Criterion on the Time History of Parameter Adjustment - Random Excitation
K = 16 q = 0

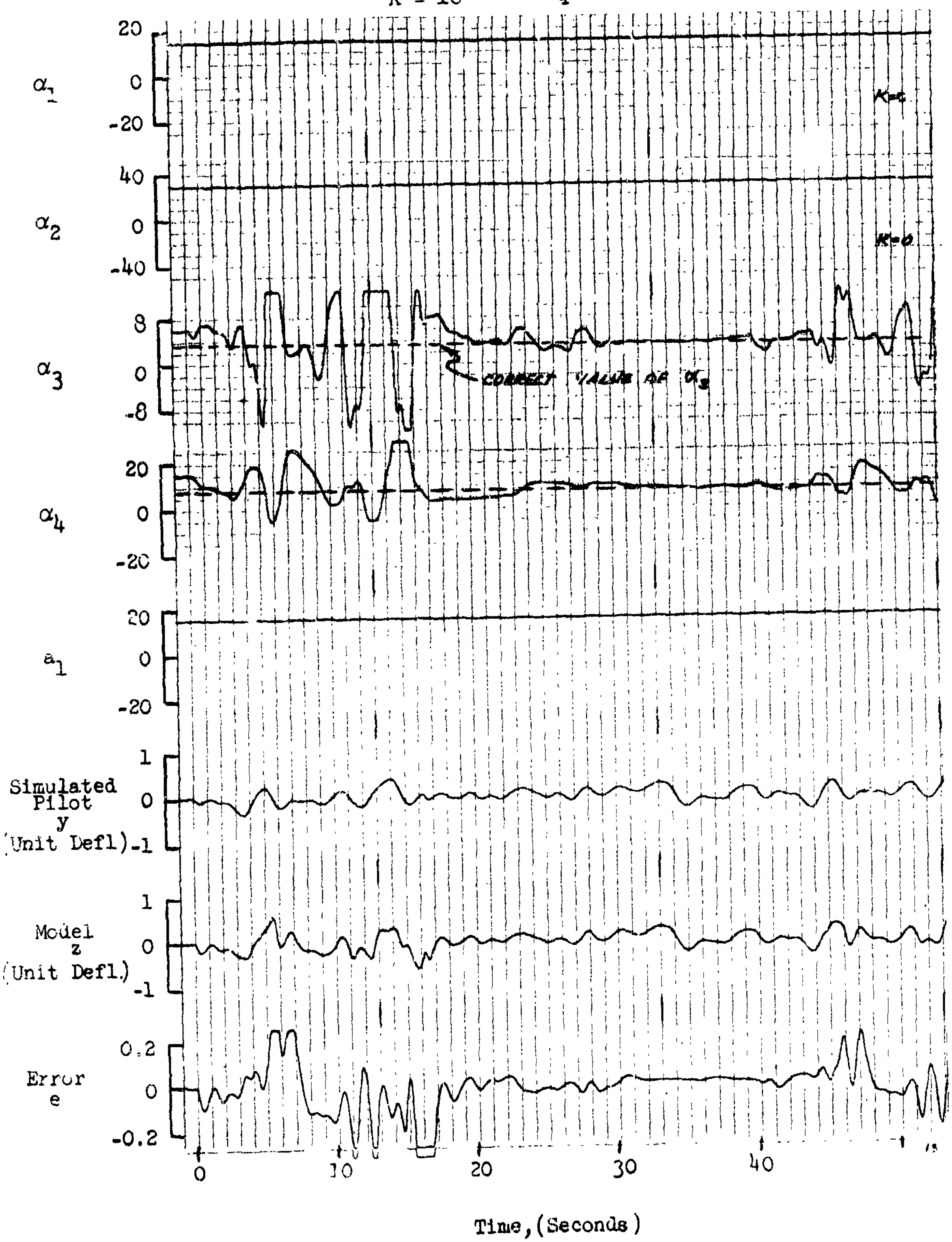


Figure 13.

Effect of Error Criterion on the Time History of
Parameter Adjustment - Random Excitation
K = 16   q = .5

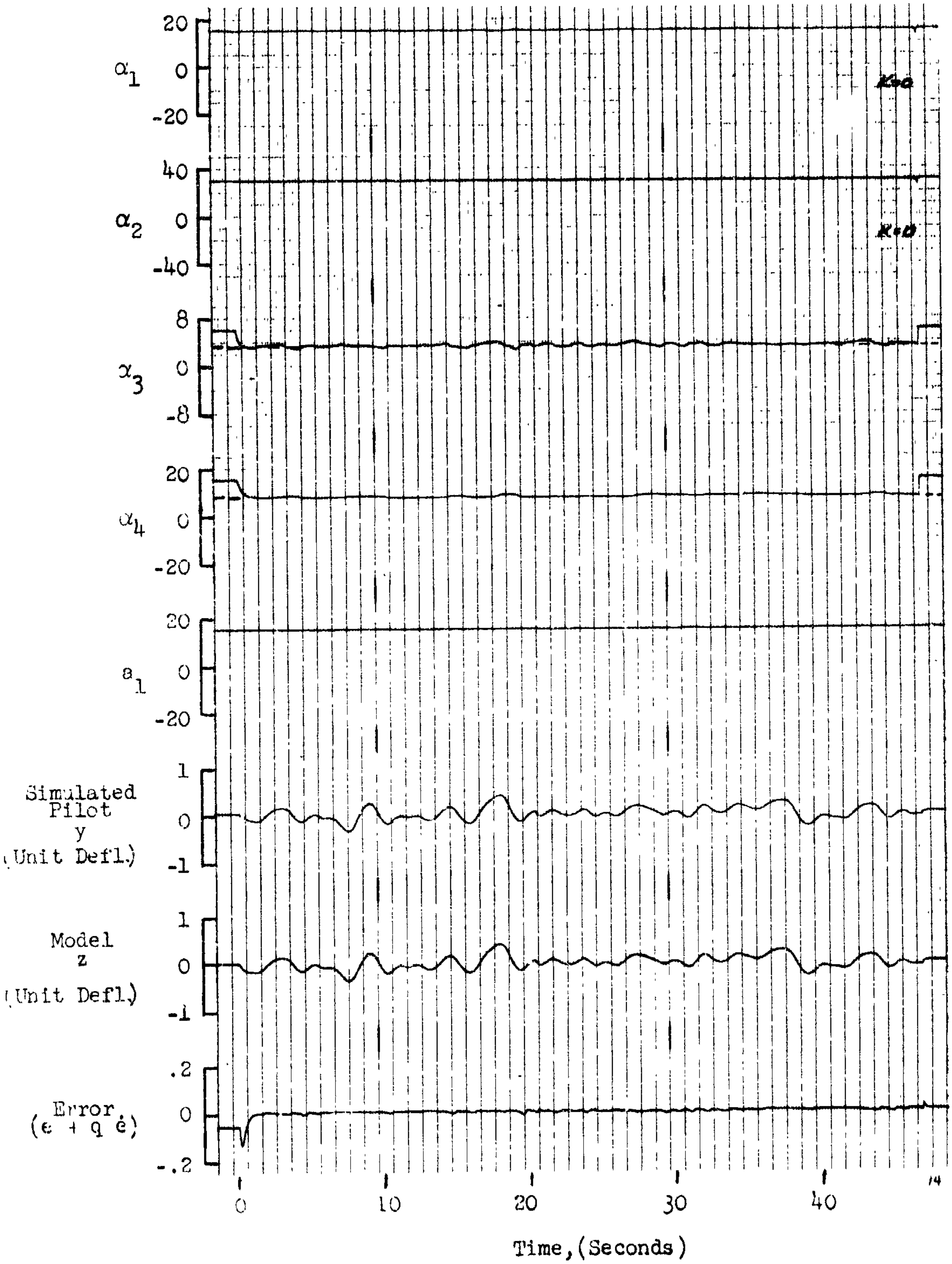


Figure 14.

Effect of Error Criterion on the Time History of
Parameter Adjustment - Random Excitation
K = 16 q = 1.0

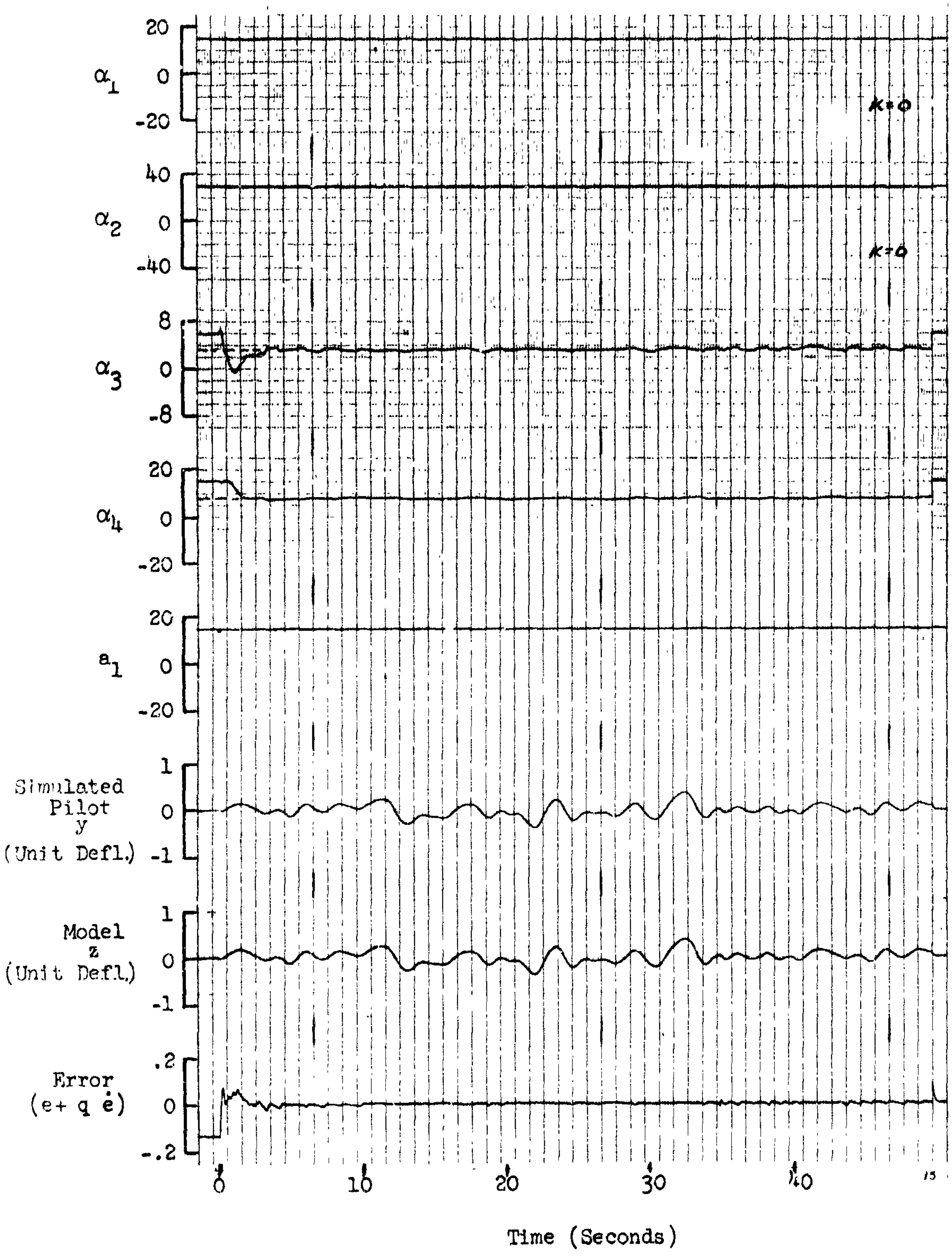


Figure 15

3.1.5 Conclusions from the Convergence Study for Fixed Parameters

This phase of the study has led to the following conclusions:

(1) The use of a modified criterion function which includes error and error rate terms (Equation 2) allows raising the adjustment gains sufficiently to obtain convergence to fixed parameter values in 1 to 3 seconds. This performance is judged satisfactory for use with time-variant parameter values.

(2) The nature of the continuous closed-loop adjustment makes the parameters sensitive to certain characteristics of the forcing function during the adjustment process. Consequently, in tracking constant or time-varying parameters random parameter excursion can be expected when both system and model are excited with a random excitation signal.

(3) The nature of the continuous model matching process produces significant levels of cross coupling between the parameters obtained.

(4) The adjustment trajectories reflect the phasing of the excitation signal and vary considerably in repeated runs performed with random excitation.

3.2 Identification of Time-Varying Parameters in a Known System

Investigations conducted under Phase II consisted of the application of the continuous model matching technique to the identification of parameters of a known time-varying system. As in Task 1 of the study the known system was chosen to be a second order differential equation, namely

$$\ddot{y} + a_1 \dot{y} + a_2 y = a_3 \dot{x} + a_4 x \qquad (3)$$

where the parameters a_1, a_2, a_3, and a_4 can be made time-variant. In particular, the system parameters were perturbed sinusoidally and stepwise, and the behavior of the corresponding model parameters α_1, α_2, α_3, and α_4

was examined. Three types of system parameter perturbations were studied on the computer:

(1) Sinusoidal variation in a parameter on the left-hand side of Equation (3), i.e., a coefficient of the dependent variable or its derivative (corresponding to a denominator term in the transfer function of an invariant system).

(2) Sinusoidal variations of a parameter on the right-hand side of Equation (3), i.e., a coefficient of one of the terms of the forcing function (corresponding to a numerator term in the transfer function of an invariant system).

(3) Step variations in the system parameters.

3.2.1 Sinusoidal Variation of Parameter α_1

Results obtained when attempting to track a sinusoidal variation of parameter a_1 are shown in Figures 16, 17, and 18. The experimental conditions imposed in each case were as follows:

Figure 16: Parameter a_1 perturbed sinusoidally at a frequency of 1.0 radians per second. Model parameters α_2, α_3, and α_4 not allowed to vary.

Figure 17: Same system parameter variation as in Figure 16 but all four parameters allowed to adjust.

Figure 18: Parameter a_1 in the system being perturbed at a frequency of 10 radians per second and all four model parameters tracking.

The criterion function used in this portion of the study did not include the rate term.

The performance of the system may be summarized by the following observations:

(1) When only the model parameter which corresponds to the perturbed parameter (in this case α_1) is allowed to adjust, an acceptable parameter tracking performance is observed. Superimposed on the sinusoidal parameter variation of α_1 are random

Time History of Model Adaptation to a Simulated Pilot's Sinusoidal Parameter, a_1
$K_1 = 20 \quad q = 0$

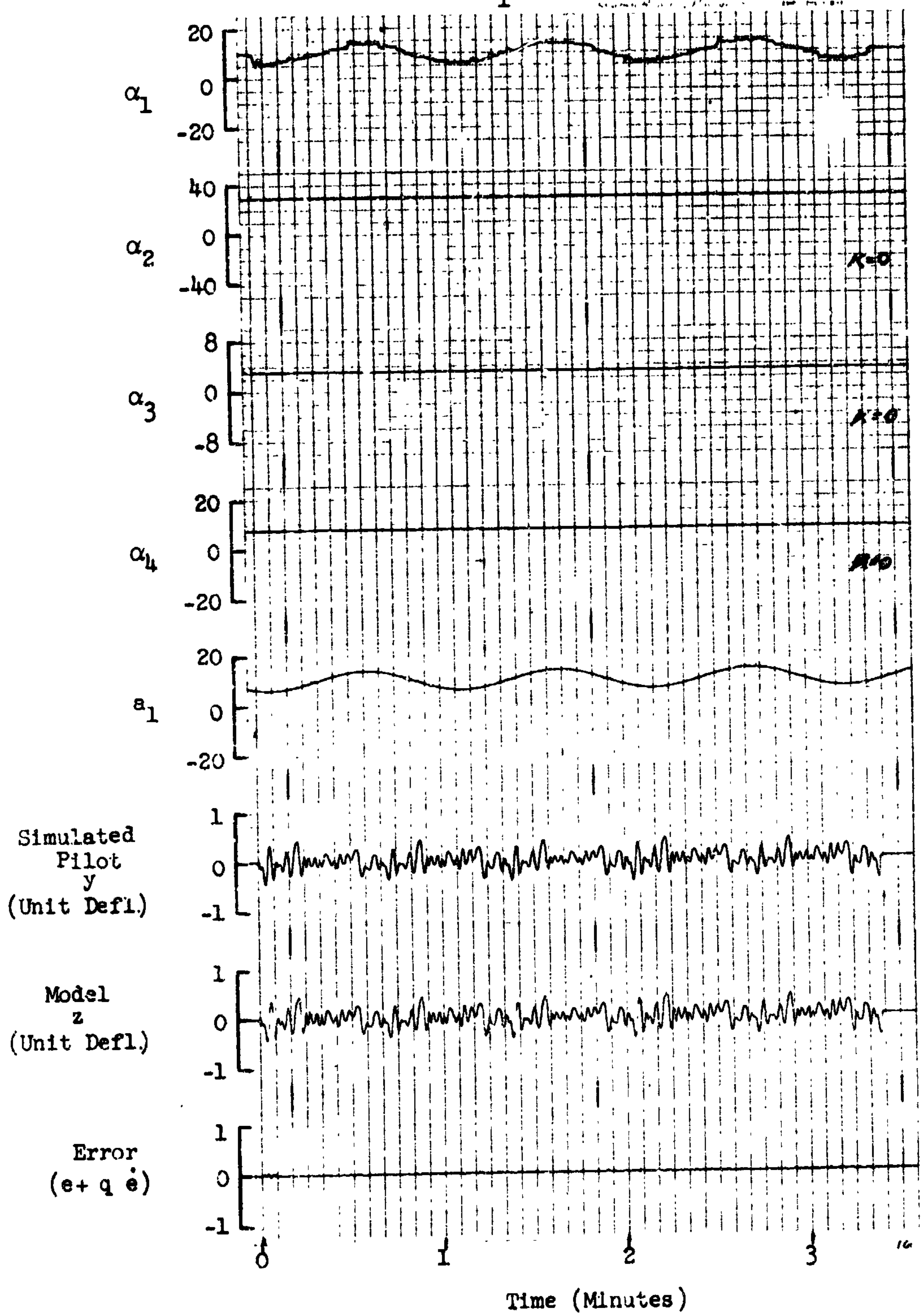


Figure 16.

Time History of Model Adaptation to a Simulated Pilot's Sinusoidal Parameter, a_1

$K = 20 \quad q = 0$

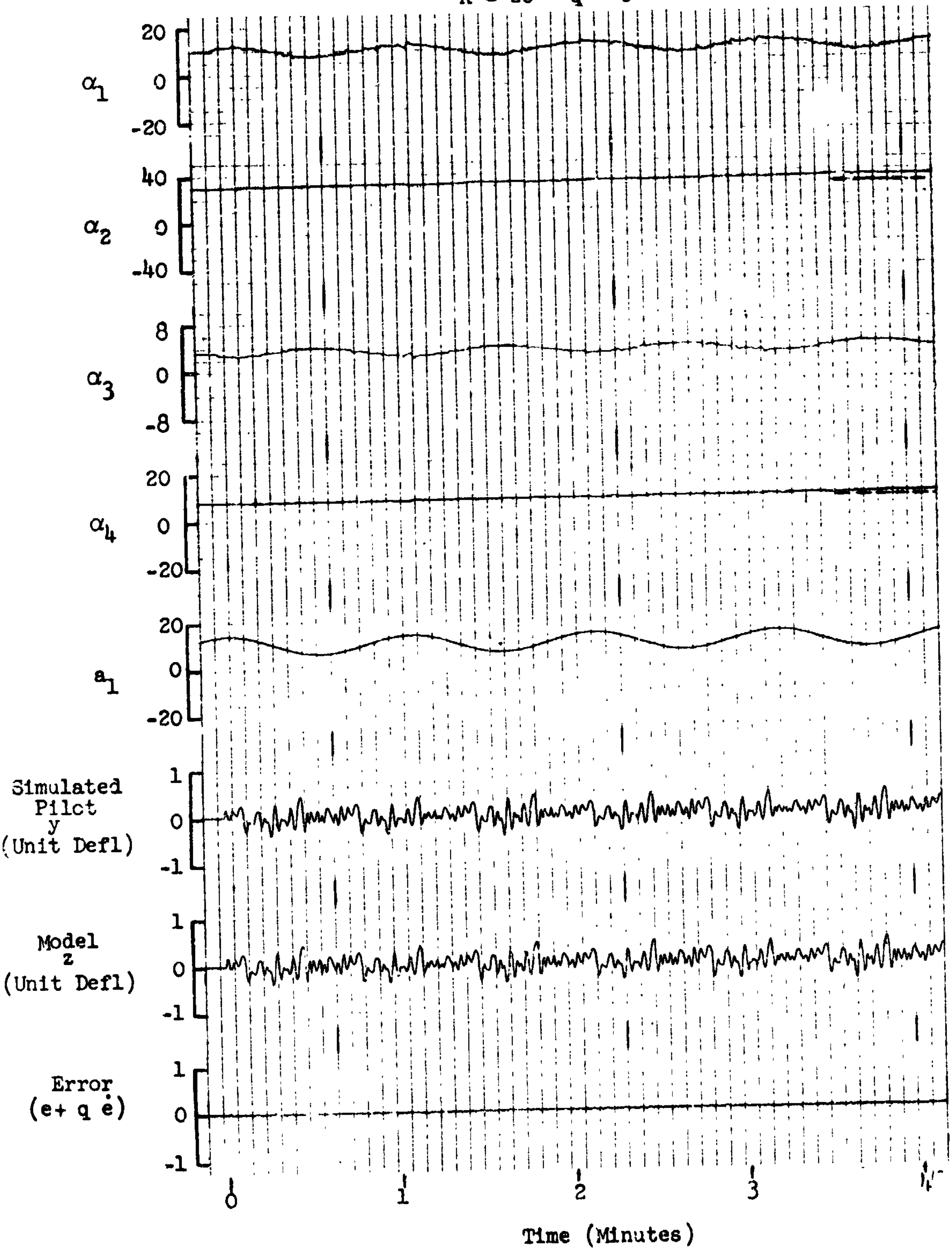


Figure 17.

Time History of Model Adaptation to a Simulated Pilot's Sinusoidal Parameter, a_1

$q = 0$

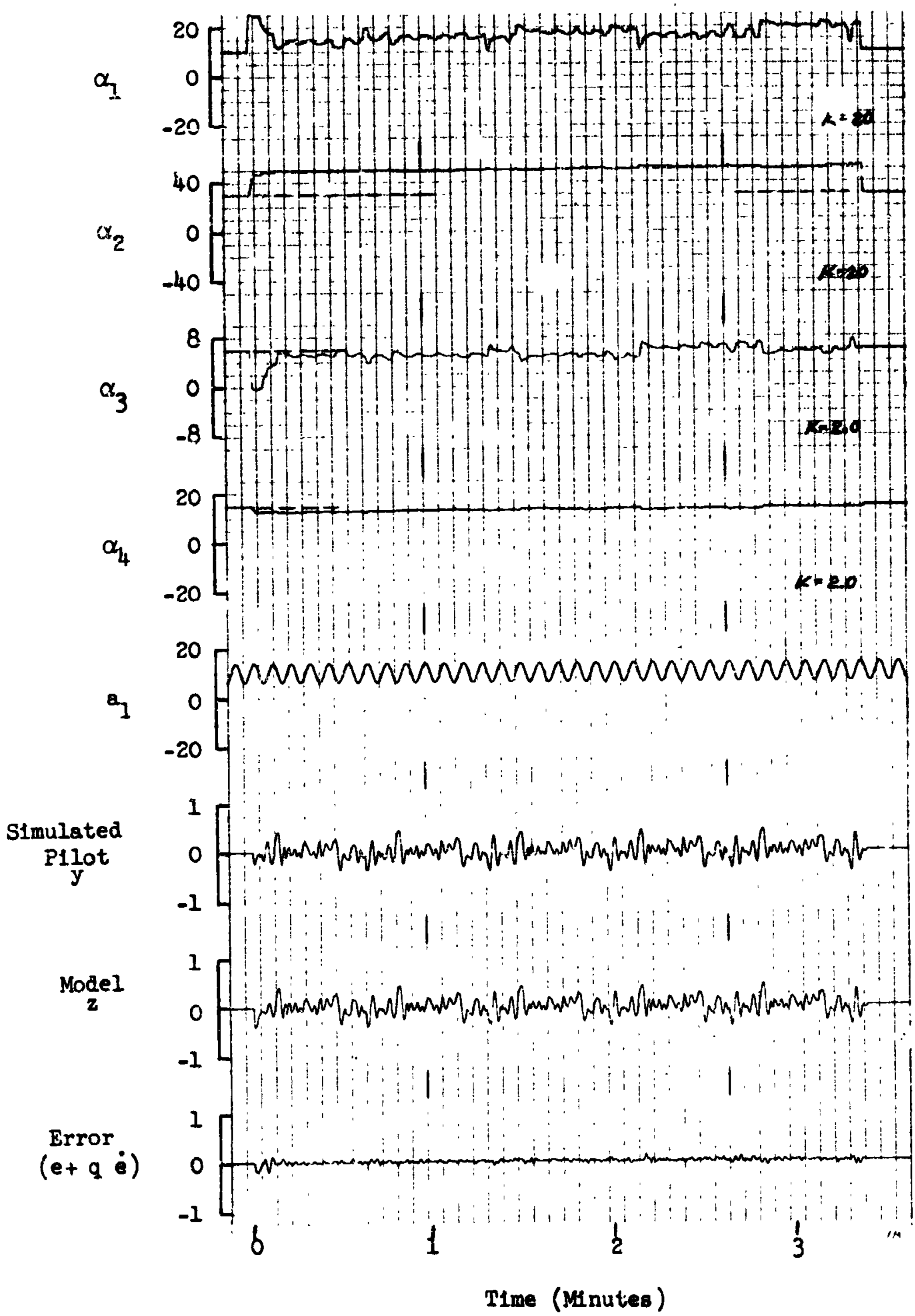


Figure 18.

components which are introduced by the random excitation signal. In Figures 16 and 17 the disturbances in the time histories of parameter α_1 and model matching error ϵ are seen to correspond to large excursions in the system and model output quantities.

(2) The results obtained when allowing all four model parameters to adjust under the condition that only one parameter (a_1) in the system is perturbed (see Figure 17) exhibit some undesirable side effects: The sinusoidal perturbation of system parameter a_1 reflects not only in the model parameter α_1 but also in parameter α_3. Furthermore, secondary cross-coupling effects are caused by α_3 variation and tend to reduce the amplitude of oscillation in parameter α_1 to a new and incorrect value. Parameters α_2 and α_4 exhibit drift from their correct values, thus indicating that their effect upon the criterion function is negligible (see Section 4).

(3) It is interesting to examine the matching errors shown in Figures 16 and 17. Figure 16 clearly presents a better match to the time variant system than Figure 17 since in the latter case parameters α_2, α_3, and α_4 vary incorrectly. Yet there is no discernible difference in the behavior of the matching error ϵ in these two cases. In other words, Figure 17 indicates that for sufficiently small model matching errors the error criterion function chosen here is too insensitive to parameter errors to cause further model adjustment.

(4) When parameter a_1 is varied at a frequency of 10 radians per second, parameters α_1 and α_3 again vary approximately in sinusoidal manner while parameter α_2 immediately assumes an incorrect value.

The frequency of the a_1-perturbation is so high that tracking behavior deteriorates, and peaks in the excitation signal are misinterpreted by the adjustment loop as belonging to the perturbation in a_1. In consequence, excitation peaks appear superimposed on the α_1-variation and also appear in α_3 due to cross-coupling. Once again, it is interesting to observe that while the parameter tracking system is not behaving satisfactorily, the matching error is extremely small, and hence is insufficient to cause further model adjustment.

3.2.2 Sinusoidal Variation of Parameter a_3

Attempts to track a sinusoidal variation of a coefficient in a forcing function term are illustrated in Figures 19, 20, and 21. During this experiment only model parameters α_3 and α_4 were allowed to vary, while parameters α_1 and α_2 were held fixed. The following comments may be made with respect to these results:

(1) Attempting to track parameter a_3 with a low value of gain ($K = 2.0$) and $q = 0$ results in the curves of Figure 19. It is apparent that parameter α_3 does not follow the sinusoidal perturbation of the corresponding parameter a_3. Furthermore, parameter α_4 drifts from its correct value to a new incorrect equilibrium position. The matching error, which is given in trace 8 of Figure 19 is quite small when the corresponding scale is taken into account.

(2) Increasing the gain to $K = 16$ with $q = 0$ results in the curves of Figure 20. Evidently both parameters α_3 and α_4 become unstable and the match between system and model becomes considerably worse. Once again, as observed in previous results, there are periods of time during which the match is rather good, followed by periods of time when the random excitation signal causes uncontrolled parameter oscillations.

(3) Figure 21 shows the effect of adding a rate term to the criterion function with all other experimental conditions remaining as in Figure 20. That is, these curves show the results of $K = 16$ and $q = 0.5$. The effect is a dramatic improvement. The match between system and model outputs is excellent and an approximately sinusoidal oscillation in α_3 is obtained. Once again the effect of random excitation peaks is reflected in random disturbances superimposed on the fundamental oscillation of parameter α_3.

Time History of Model Adaptation to a Simulated
Pilot's Sinusoidal Parameter, a_3
$q = 0$

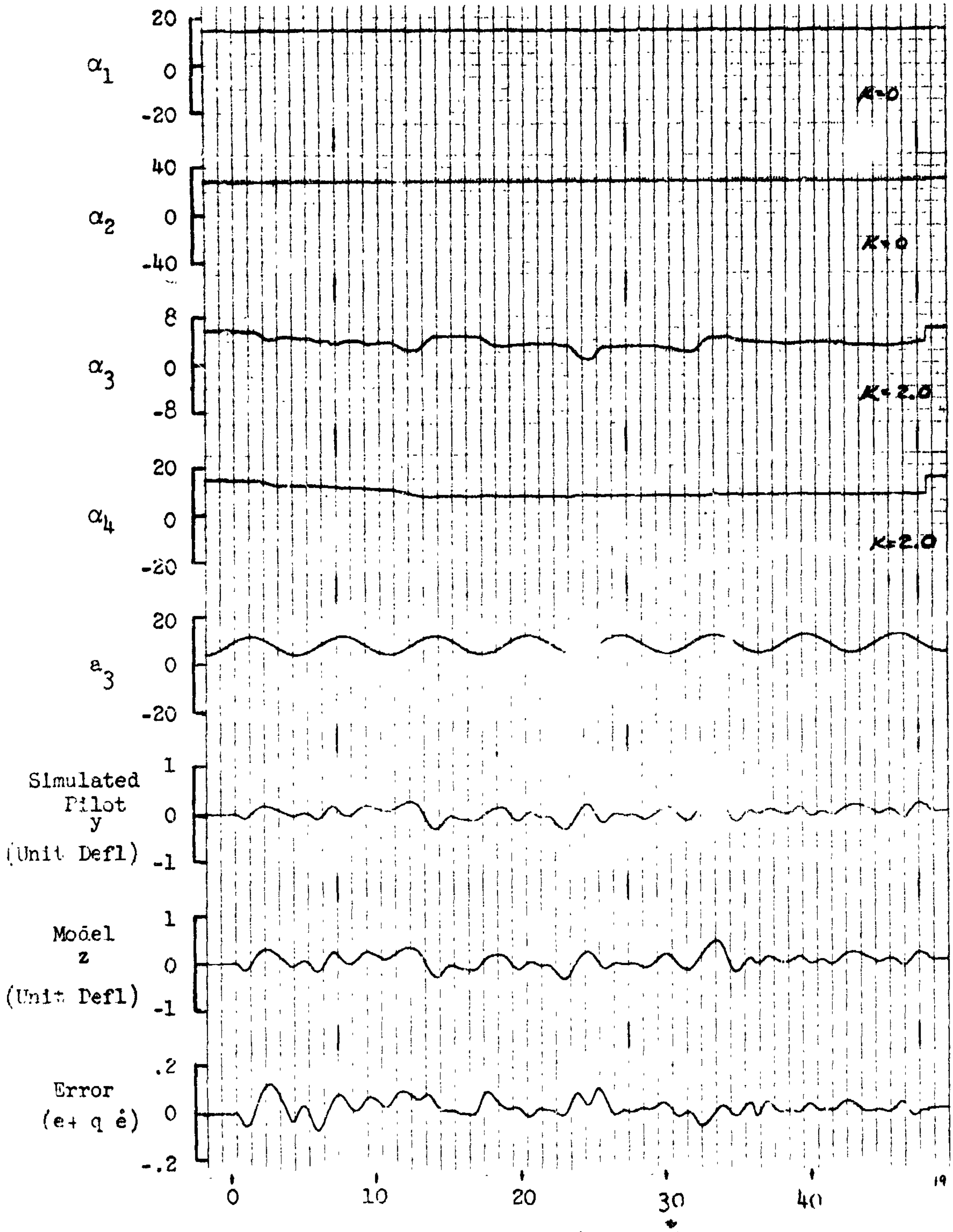


Figure 19.

Time History of Model Adaptation to a Simulated Pilot's Sinusoidal Parameter, a_3
$q = 0$

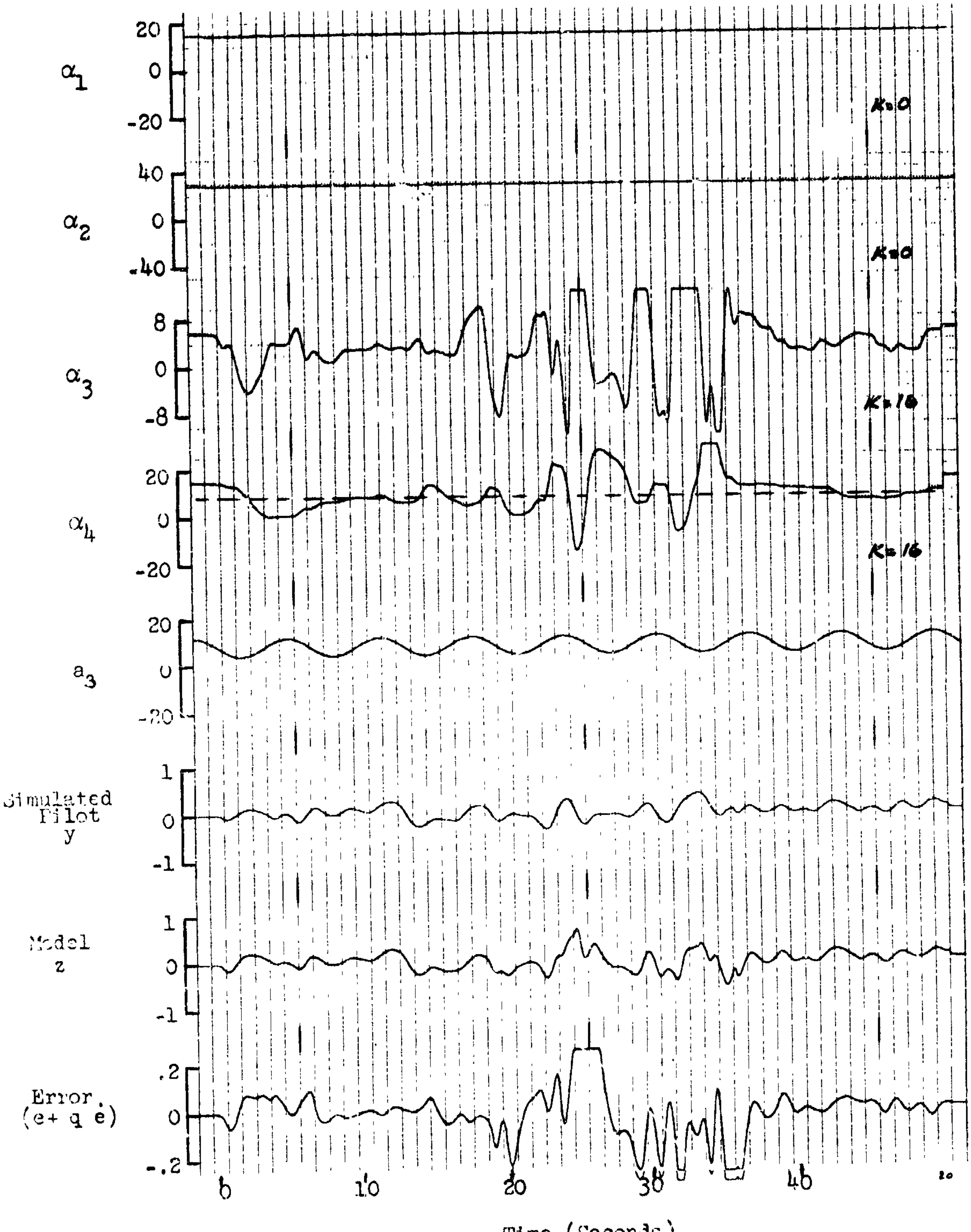


Figure 20.

Time-History of Model Adaptation of a Simulated Pilot's Sinusoidal Parameter, a_3.

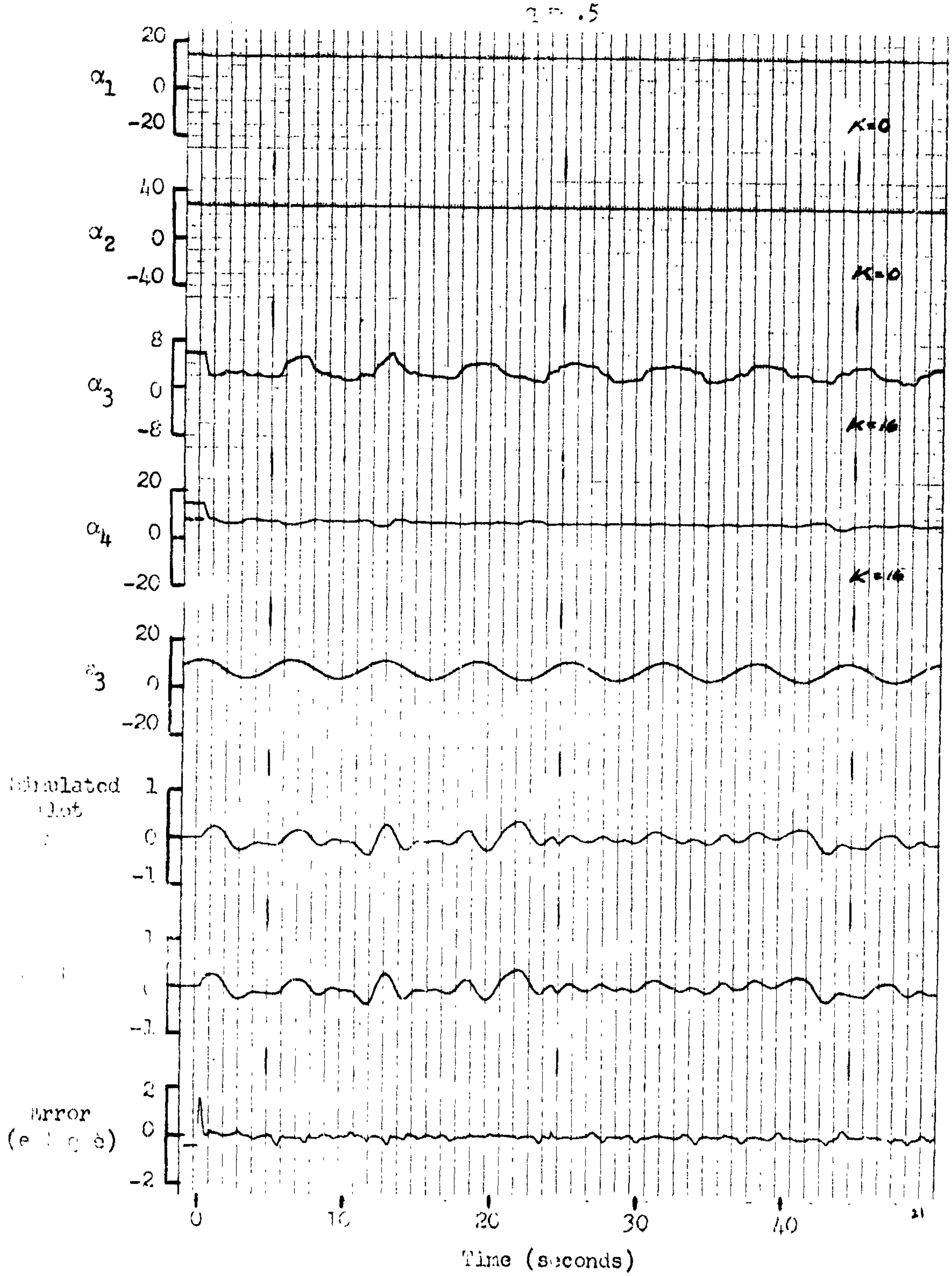


Figure 21

3.2.3 Step Variations of Parameter a_3

Figures 22 and 23 show the behavior of the parameter tracking system when parameter a_3 is perturbed with step changes at a low and high frequency respectively. The adjustment gain is $K = 16$, and $q = 1.0$. All four parameters are allowed to track. Consider first Figure 22 which in effect corresponds to the behavior of the model matching technique for fixed parameters since adequate time for parameter adjustment elapses before the values are changed. The match between system output and model output is excellent but a switching transient is observed in the matching error. This is particularly true in trace 8 which includes the effect of rate of change of error.

Consider now Figure 23 where parameter a_3 is perturbed by a square wave signal. It can be seen that the system and model outputs contain significant energy at frequencies approximately equal to the fundamental frequency of the square wave. Consequently, the behavior of parameter α_3 in the model varies from cycle to cycle, depending on the corresponding initial conditions present in the random excitation signal. Cross-coupling of parameters is again evident, both in α_1 and in α_4.

3.2.4 Conclusions from the System Parameter Variation Study

The major conclusion from this phase of the study is that continuous model matching techniques can indeed be applied to the tracking of time-varying parameters in a known system. Satisfactory tracking of coefficients of the dependent variables as well as coefficients of the forcing function terms can be accomplished, provided that the modified criterion function given by Equation (2) is used.

3.3 Identification of Time-Variant Human Operator Parameters

Following the completion of Phase II an attempt was made to use the continuous model matching technique to identify the parameters of a human operator in a tracking task so constructed that the operator's behavior became time-varying. The operator's response presumably adjusts to changes in the controlled elements. The controlled element gain and "time constants" were varied as functions of time as outlined in Section 2. The results for two different operators performing the same time-varying tasks are shown in Figures 24 and 25.

Time-History of Model Adaptation to a Simulated Pilot's Stepped Parameter, a_3

k = 16 q = 1.0

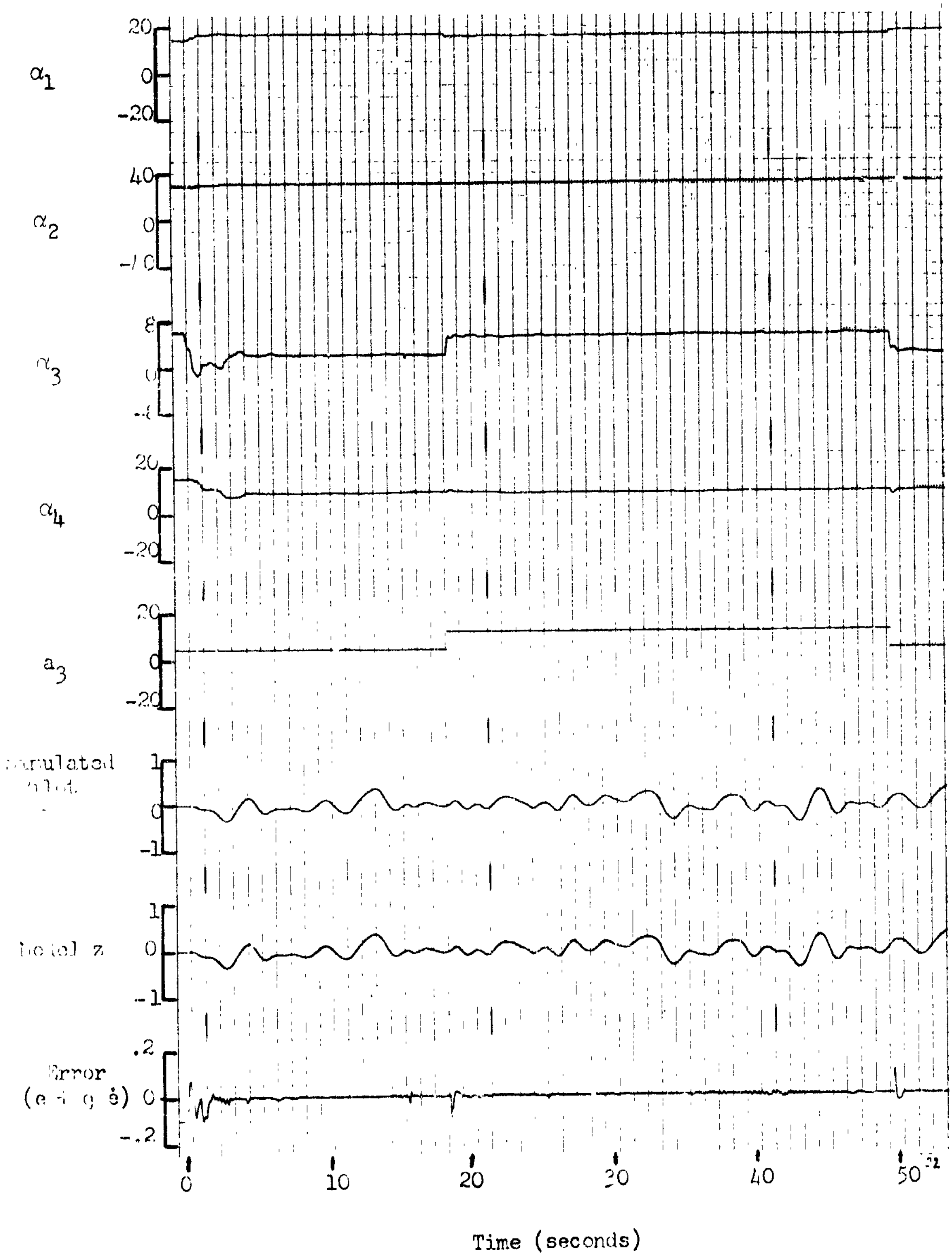


Figure 22

Time-History of Model Adaptation to a Simulated Pilot's Stepped Parameter, a_3.

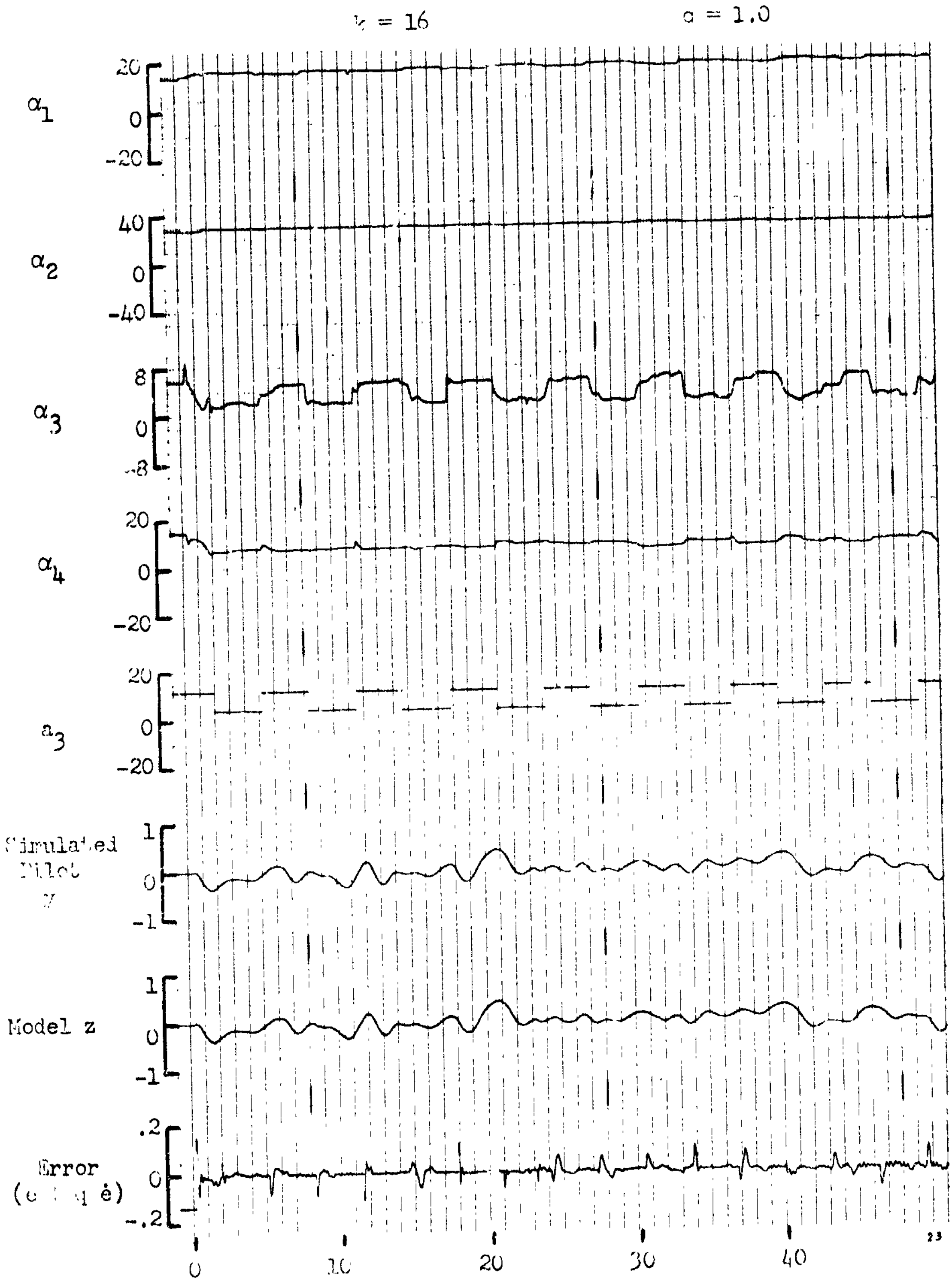


Figure 23

Consider first Figures 24a and b. Parameter α_1 does not reveal any well-defined pattern and can be considered approximately constant for the five-minute duration of the run. Parameter α_2 exhibits what appear to be significant changes. As the plant gain is increased parameter α_2 likewise increases while parameter α_4 decreases. As the plant is transformed into a double integrator, parameter α_2 further increases while parameter α_4 further decreases. The trend observed in parameter α_2 is essentially reversed in α_3 in the course of these plant variations. Effects of these changes manifest themselves in the tracking behavior, as well as in the mathematical model. An examination of the second trace of Figure 24b, the operator's output, shows that during the portions of the run when the loop gain was high the amplitude of the operator's corrections was correspondingly lower. This behavior is clearly expected since the same magnitude of correction can be obtained with a smaller stick displacement when the plant gain is higher.

The effect of the observed time variations in parameters α_2, α_3 and α_4 is most clearly discernible in terms of the gain and lead time constant in the operator's mathematical model, if it is assumed that toward the end of each phase of the tracking run an invariant model is approximately valid. Average values of the parameters α_1 through α_4 were read at the times indicated in Figure 24a. The values of model gain K_1 and model lead time constant τ_n obtained at these four values of time are given below in Table 1.

Indicated Time	Model Gain K_1	Lead Time Const. τ_n
t_1	.437	1.5
t_2	.278	1.9
t_3	.150	2.8
t_4	.443	1.5

Table 1.

Time-History of Human Pilot Compensatory Tracking
in a Time Varying Task

Pilot "R"

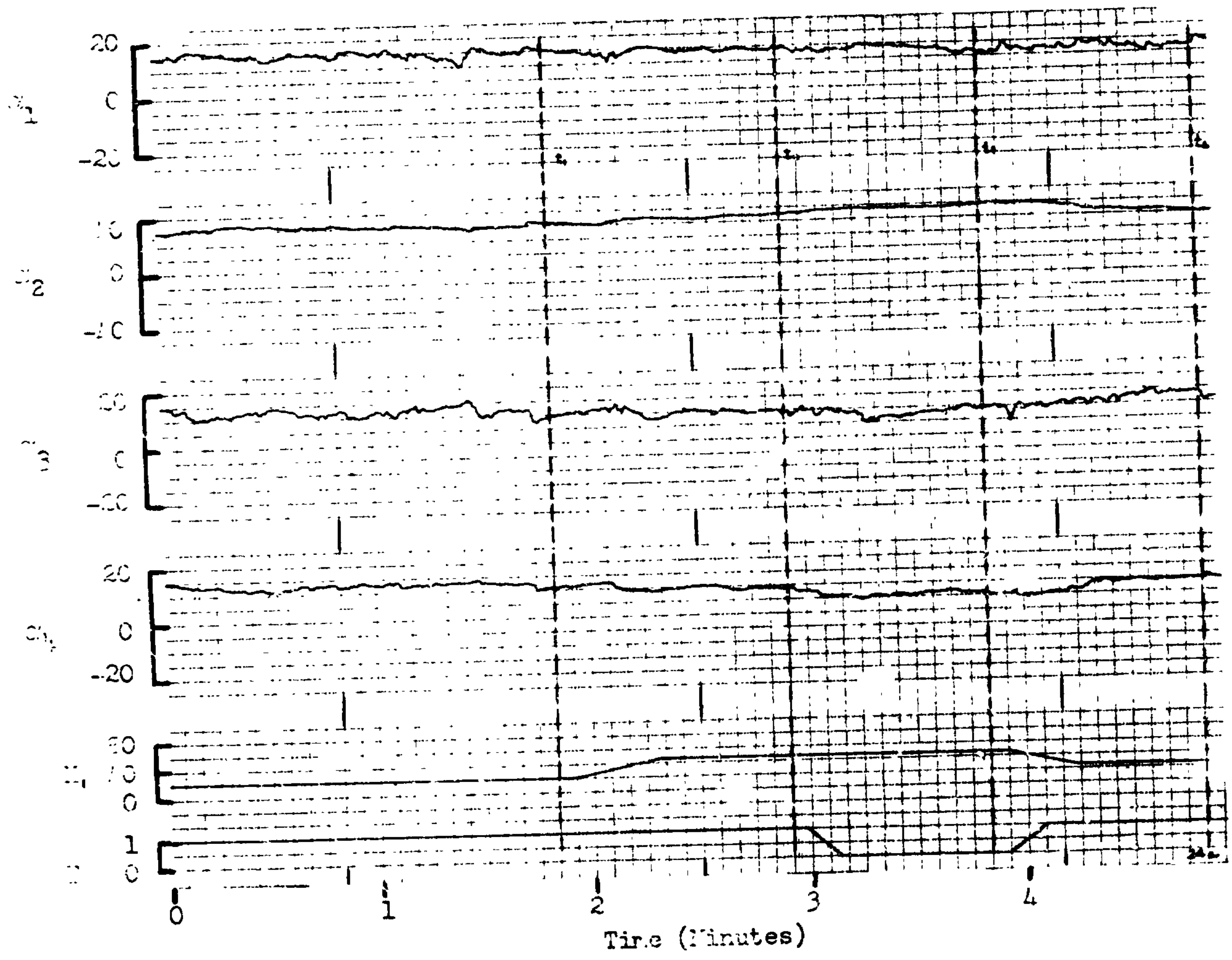


Figure 24a

Time-History of Human Pilot Compensatory Tracking

in a Time Varying Task

Pilot "R"

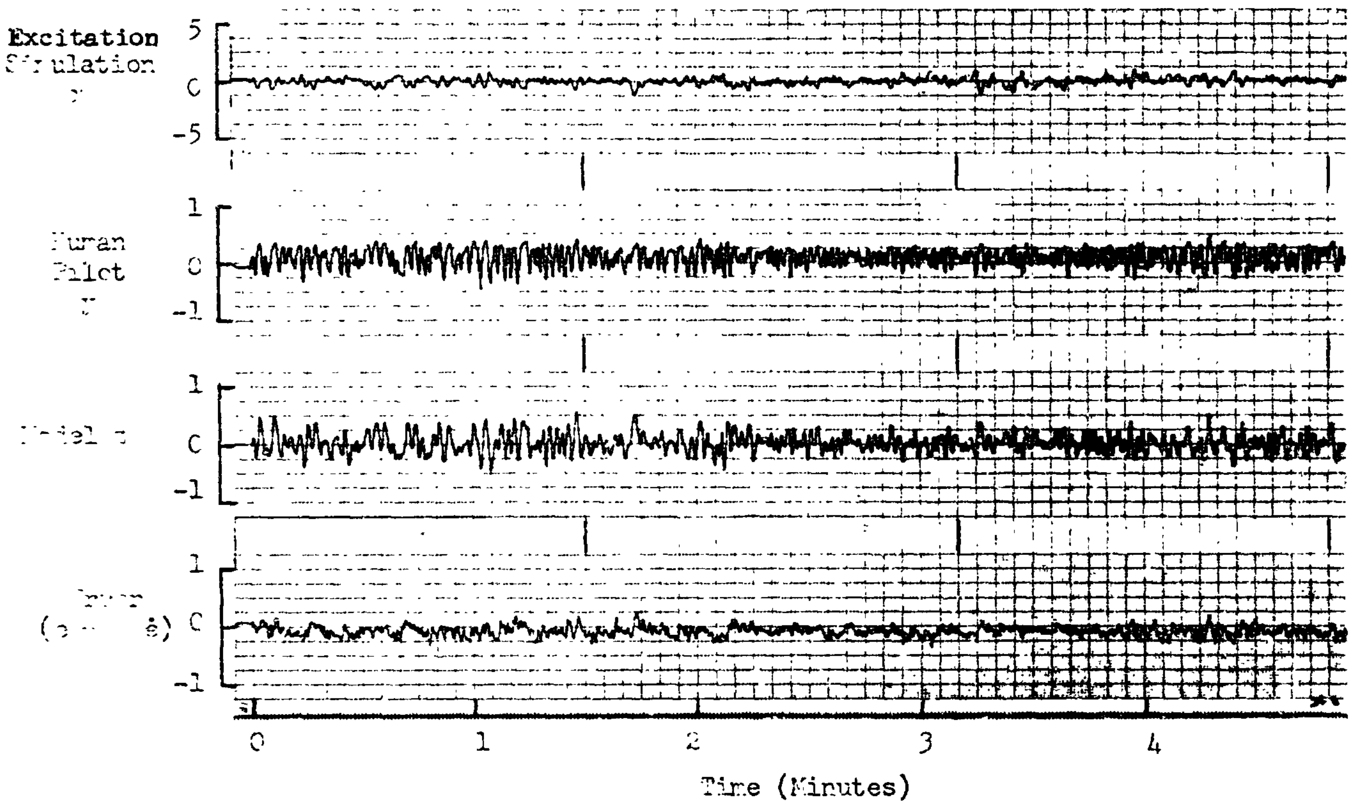


Figure 24b

Time-History of Human Pilot Compensatory Tracking in a Time Varying Task

Pilot "B"

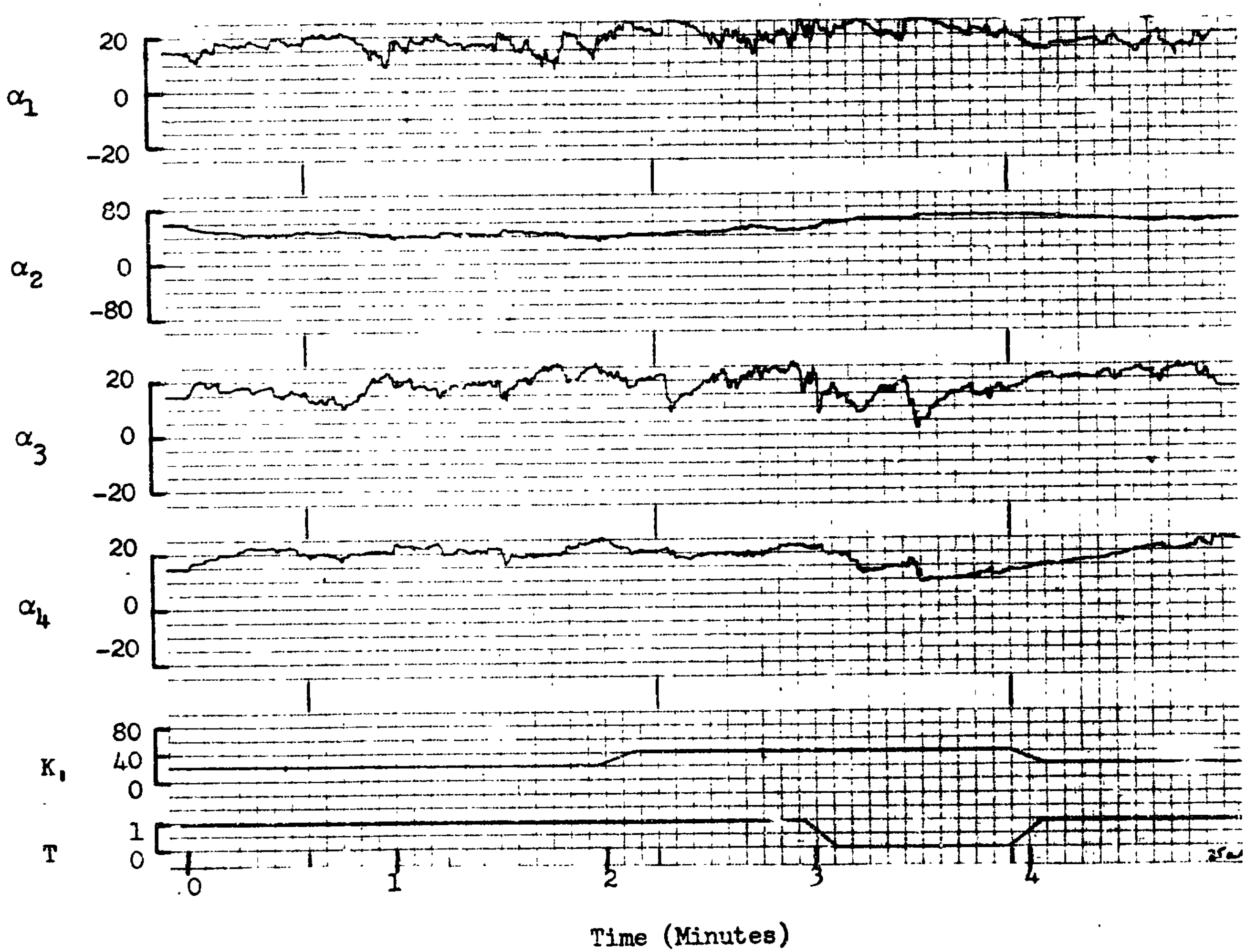


Figure 25a

Time-History of Human Pilot Compensatory Tracking in a Time Varying Task

Pilot 'B'

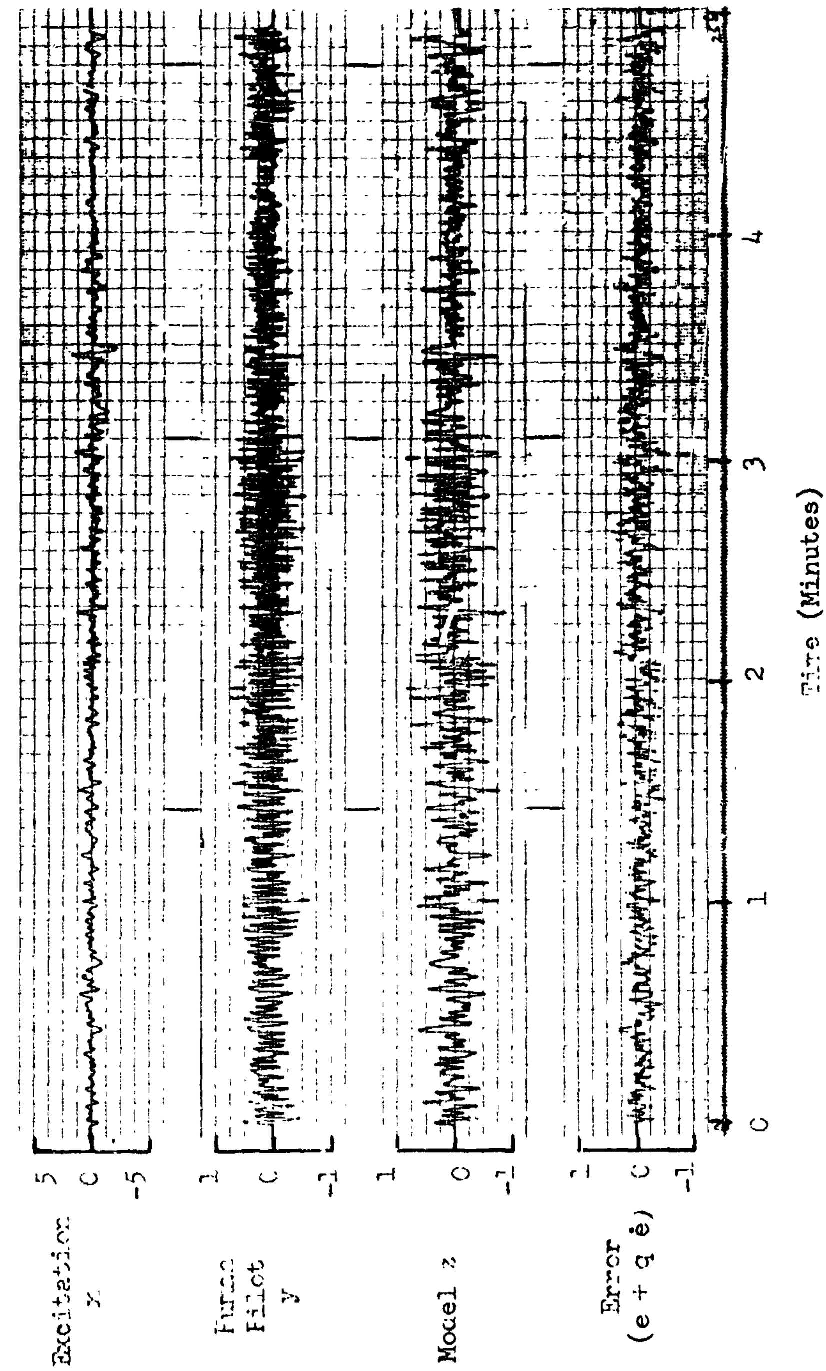


Figure 25b

It can be seen from an inspection of this table that as the difficulty of the task increases (the plant gain increases and the plant is changed to a double integration) the operator's gain decreases and his lead time constant increases. In other words, the operator increases his attention to signal extrapolation, and he does this at the expense of gain. It is interesting to note from Figure 24a that the matching error is smallest when the plant gain is highest and when the plant is a double integrator. As the plant is once again adjusted to its original condition the model parameters return to approximately their original values and the matching error again approaches its original value.

A similar pattern of behavior is observable in the records of Figures 25a and 25b, with the qualification that the operator performing the task in this case exhibits a considerably greater variation in response than the previous one as is indicated by larger excursions in the parameter values. As before no apparent pattern in the adjustment of parameters α_1 and α_3 is observable. However, parameter α_2 does show an apparently significant variation, as does parameter α_4. Other observations made in connection with Figure 24 apply similarly in this case. Thus, the operator's output decreases in amplitude when the plant gain increases, and the matching error becomes smaller. It is expected that longer training times would have resulted in smoother performance for both operators.

3.4 Conclusions

The results of the study show the feasibility of using continuous parameter tracking techniques for the identification of time-varying human pilot parameters. However, considerable caution must be exercised in interpreting particular parameter values in a time-varying model since these values may be caused by a combination of a number of factors including the effect of the excitation signal, transients caused by particular initial conditions, and cross-coupling between parameters.

4. ANALYTICAL CONSIDERATIONS

4.1 Cross-Coupling Effects in Parameter Adjustment

The observation of cross-coupling effects which occur during simultaneous adjustment of several parameters (see for example, Section 3.2.1) suggests a more detailed analysis of the underlying mathematical relations. Such an investigation is desirable in order to gain a better understanding of the nature of multi-dimensional parameter adjustment and to determine whether certain transient phenomena observed in the computer study are inherent in the adjustment procedure or are caused by computer inaccuracy.

Considering the steepest descent equation (for $q = 0$)

$$\dot{\alpha}_i = -K \frac{\partial F}{\partial \alpha_i} = -K \epsilon \frac{\partial z}{\partial \alpha_i} \qquad (4)$$

it can be seen that the adjustment rate is proportional to the model matching error ϵ and the sensitivity $\partial z / \partial \alpha_i$. In this section the sensitivity coefficients will be denoted by $\partial z / \partial \alpha_i = u_i$, using only one subscript for convenience.* The error term ϵ may be expanded, in first order approximation, in terms of individual parameter errors $\Delta\alpha_i$, viz.,

$$\epsilon \cong u_1 \Delta\alpha_1 + \cdots + u_4 \Delta\alpha_4 \qquad (5)$$

where higher order effects, noise, and uncertainty in the structure of the mathematical model are omitted. This equation implies $\epsilon = 0$ when all parameters α_i have been adjusted to the desired values a_i, such that $\Delta\alpha_i = 0$. Combining Equations (4) and (5) one obtains

$$\dot{\alpha}_i = -K u_i \sum_{j=1}^{4} u_j \Delta\alpha_j \qquad (6)$$

Hence each parameter adjustment rate $\dot{\alpha}_i$ is sensitive, to a varying degree, to all of the instantaneous parameter adjustment errors $\Delta\alpha_j$. This sensitivity is expressed by the (approximate) square matrix (S) with time-varying

---

*Thus the notation formerly used will be abridged as follows:

$$\frac{\partial z}{\partial \alpha_1} = u_{11} \triangleq u_1, \quad \cdots, \quad \frac{\partial z}{\partial \alpha_4} = u_{14} \triangleq u_4$$

$$\frac{d u_{11}}{dt} = u_{21} \triangleq \dot{u}_1, \quad \cdots, \quad \frac{d u_{14}}{dt} = u_{24} \triangleq \dot{u}_4 .$$

elements

$$S_{ij} = u_i u_j \qquad (7)$$

such that

$$(\dot{\alpha}) = (S)(\Delta\alpha) \qquad (8)$$

The off-diagonal elements of the matrix are the cross-coupling coefficients. Under random excitation x(t) the cross-coupling terms tend to have small average values, provided the u_i and u_j terms are statistically independent. The diagonal terms u_i^2 are non-negative and their average values are always larger than those of the cross-coupling terms. It is noted that according to Equation (7) the matrix is symmetric.

Inspection of the time-histories exhibited in Figures 26 and 27 reveals that the sensitivity terms u_1 and u_3 dominate u_2 and u_4, respectively. This is explained by the fact that u_1 and u_3 are obtained as solutions of sensitivity equations in which the time-derivatives $\dot{z}$ and $\dot{x}$ rather than z and x are the forcing functions (see Equations (I-10) and (I-12), page 59, of the Task I Report, Ref. 1). Considering the frequency content of the excitation signal x and the dependent variable z it follows that $\dot{x}$ and $\dot{z}$ have larger maximum excursions than x and z, respectively. (Additional analysis will be presented in Section 4.2.)

These facts explain the prevalence of cross-coupling effects between errors in the α_1 and α_3 adjustments which were observed in Figures 17 and 18. In these instances the variation in α_1 which was caused by sinusoidal perturbation of the corresponding system parameter a_1 also produced a sinusoidal variation in α_3. This effect in turn caused secondary cross-coupling in α_1.

Cross-coupling effects are also noticed when all but one model parameter are initially set at their correct values. During the adjustment of the initially incorrect parameter some transients will also occur in the remaining parameters α_i as a result of cross-coupling, according to Equation (6). This is true because the sensitivities u_i corresponding to these parameters are not precisely zero due to small computational errors.

4.2 Functional Relation Between Sensitivity Coefficients

A closer examination of the sensitivity coefficients u_i and their interrelation is of interest to obtain quantitative estimates of relative magnitudes.

Time History of the Influence Coefficients
(Corresponds to Figure 13)

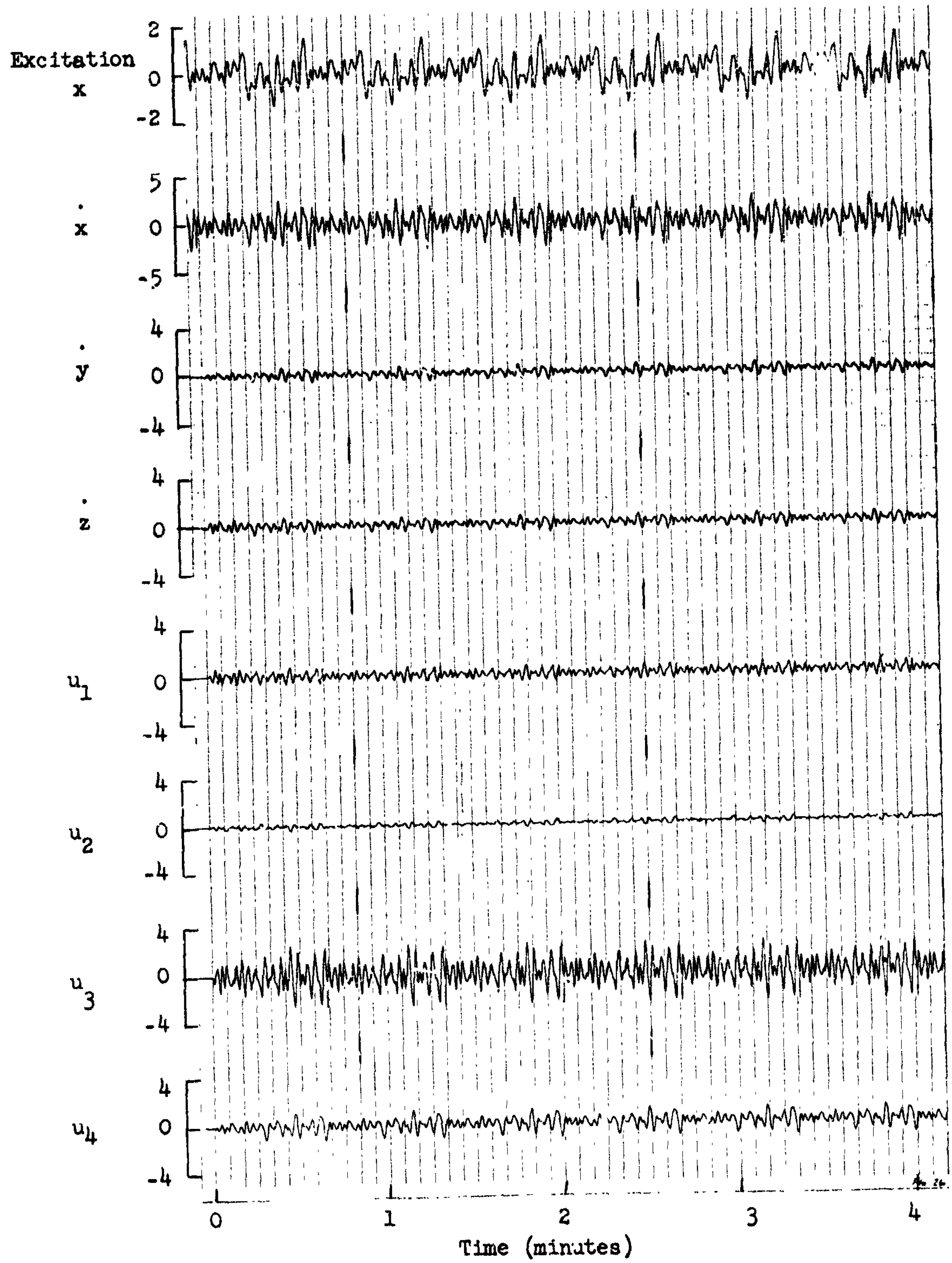


Figure 26.

Time History of the Influence Coefficients

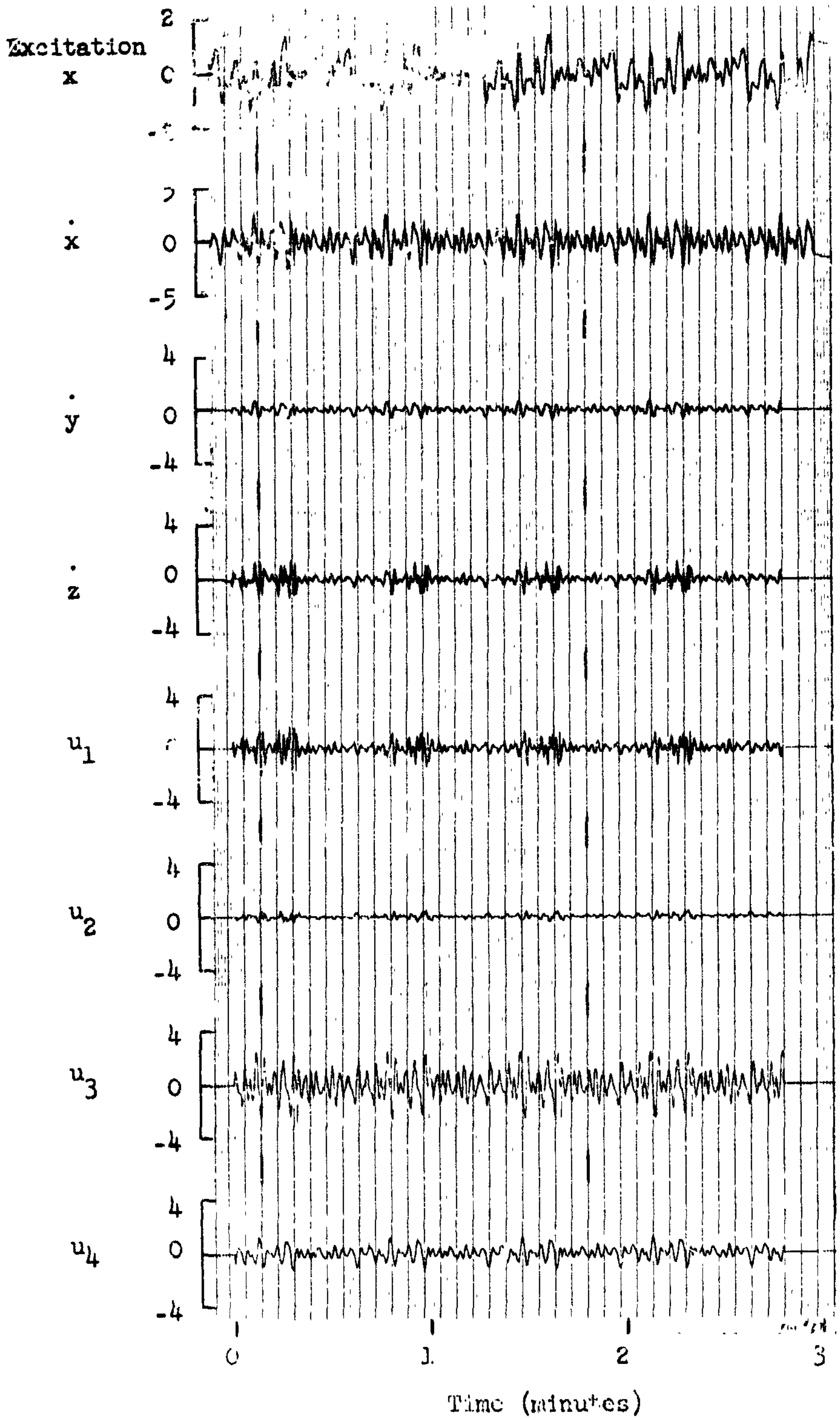


Figure 27.

Consider the sensitivity equations for u_1 and u_2 (see Task I Report, Page 59) for invariant α_1, α_2

$$\ddot{u}_1 + \alpha_1 \dot{u}_1 + \alpha_2 u_1 = -\dot{z} \qquad (9)$$

$$\ddot{u}_2 + \alpha_1 \dot{u}_2 + \alpha_2 u_2 = -z \qquad (10)$$

with initial values $u_1(0) = \dot{u}_1(0) = u_2(0) = \dot{u}_2(0) = 0$.
Time-differentiation of Equation (10) yields the approximate relation

$$u_1 \cong \dot{u}_2 \qquad (11)$$

Transient differences between u_1 and $\dot{u}_2$ are caused by a non-zero initial value

$$\ddot{u}_2(0) = -z(0)$$

Note that $\ddot{u}_1(0)$ equals zero by definition but $\ddot{u}_2(0)$ in general does not equal zero.

Similarly, the corresponding two sensitivity equations for u_3 and u_4

$$\ddot{u}_3 + \alpha_1 \dot{u}_3 + \alpha_2 u_3 = \dot{x} \qquad (12)$$

$$\ddot{u}_4 + \alpha_1 \dot{u}_4 + \alpha_2 u_4 = x \qquad (13)$$

with initial values $u_3(0) = \dot{u}_3(0) = u_4(0) = \dot{u}_4(0) = 0$
yield the approximate relation

$$u_3 \cong \dot{u}_4 \qquad (14)$$

which is valid after transient differences between u_3 and $\dot{u}_4$ due to $x(0) \neq 0$ have subsided.

Combination of the sensitivity equations (11), (13) and the original model equation

$$\ddot{z} + \alpha_1 \dot{z} + \alpha_2 z = x \qquad (15)$$

yields the approximate relation

$$u_2 x = -u_4 z \qquad (16)$$

which is applicable after transients due to $z(0)$ and $\dot{z}(0)$ have disappeared.

It is important to note that Equations (11), (14) and (16) imply time-invariant coefficients. If α_1 and α_2 are time-variant, a time-differentiation of Equation (10) yields:

$$\dddot{u}_2 + \alpha_1 \ddot{u}_2 + \alpha_2 \dot{u}_2 = -z - \dot{\alpha}_1 \dot{u}_2 - \dot{\alpha}_2 u_2 \qquad (17)$$

In this case a direct equivalence with Equation (9) leading to the approximation $u_1 \cong \dot{u}_2$ can no longer be established. Nevertheless it is reasonable to assume that for sufficiently small adjustment rates $\dot{\alpha}_1$, $\dot{\alpha}_2$, the approximations (11) and (14) are still useful in providing estimates of the relative magnitudes of the u_i terms.

In some applications it should also be of interest to consider the very considerable simplification which results from using the approximate relations (11) and (14) in implementing the parameter matching system on the computer. This computer program would include only Equations (10) and (13) yielding u_1, u_2 and u_3, u_4, respectively. A similar formulation has been used by W. J. Klenk in simplifying the computer implementation of an adaptive control system, where two distinct parameter influence coefficients were obtained from a single sensitivity equation (see Reference 3).

4.3 Precision of Parameter Matching

As was observed in the discussion of the computer results, different parameters of the system are matched with different degrees of precision. The relative magnitude of the sensitivities u_i helps to explain this fact. Equation (5) shows that the instantaneous model matching error constitutes a weighted average of the individual adjustment errors $\Delta\alpha_i$ where u_i are the weighting factors. Clearly those adjustment errors which are characterized by dominant weighting factors will be adjusted with the greatest precision, and vice versa. Since u_3 dominates in many of the cases examined, it is not surprising to find the precision of the α_3 adjustment is quite high. By contrast, u_2 is dominated by the other sensitivities and hence α_2 is poorly defined. These results are also borne out by Figures 5, 6, 7 of the Task I Report (pages 19-21).

Further investigation of the underlying mathematical relations will clarify the picture. The following discussion applies rigorously only to the time-invariant case, but serves to explain basic trends in the time-variant case as well.

It is first observed that the random excitation signal $x(t)$ includes frequencies up to 5 rad/sec due to human tracking behavior. Therefore, $\dot{x}$ has higher amplitudes in the upper frequency band than x, and causes u_3 to dominate u_4 (see Equations (12), (13), (14)). Similarly, since z and $\dot{z}$

reflect x and $\dot{x}$, it is expected that u_1 will dominate u_2, depending on the filtering characteristics of the model equation (15). Typical values of transfer function gain for the time-invariant system over the range of input frequencies are

$$\left|\frac{Z}{X}\right| = 0.15 \ldots 0.25$$

when X and Z are Laplace transforms of x, z. Furthermore, an estimate of u_2 and u_4 magnitudes can be obtained on the basis of Equation (16), viz.,

$$\left|\frac{U_2}{U_4}\right| \cong \left|\frac{Z}{X}\right|$$

Therefore, the sensitivity term u_4 dominates u_2 at least by a ratio of 3:1. This result was confirmed in many of the time histories obtained in Section 3. For the conditions under which the model matching system was operated it may be seen that u_3 dominates u_4 which in turn dominates u_2. Also, u_1 dominates u_2, hence the poor definition of α_2 and the generally good definition of α_3 observed in many of the computer runs. The relatively larger values of u_3 also tend to make the α_3 adjustment loop the most critical in terms of stability as was observed in Section 3.1.4.

It is interesting to note that the qualitative discussion derived for the time-invariant case still applies in the time-variant case considered here. However, further investigation along these lines will be very desirable. This will also have to include the effect of the rate stabilization term $q\dot{\epsilon}$ in the error criterion.

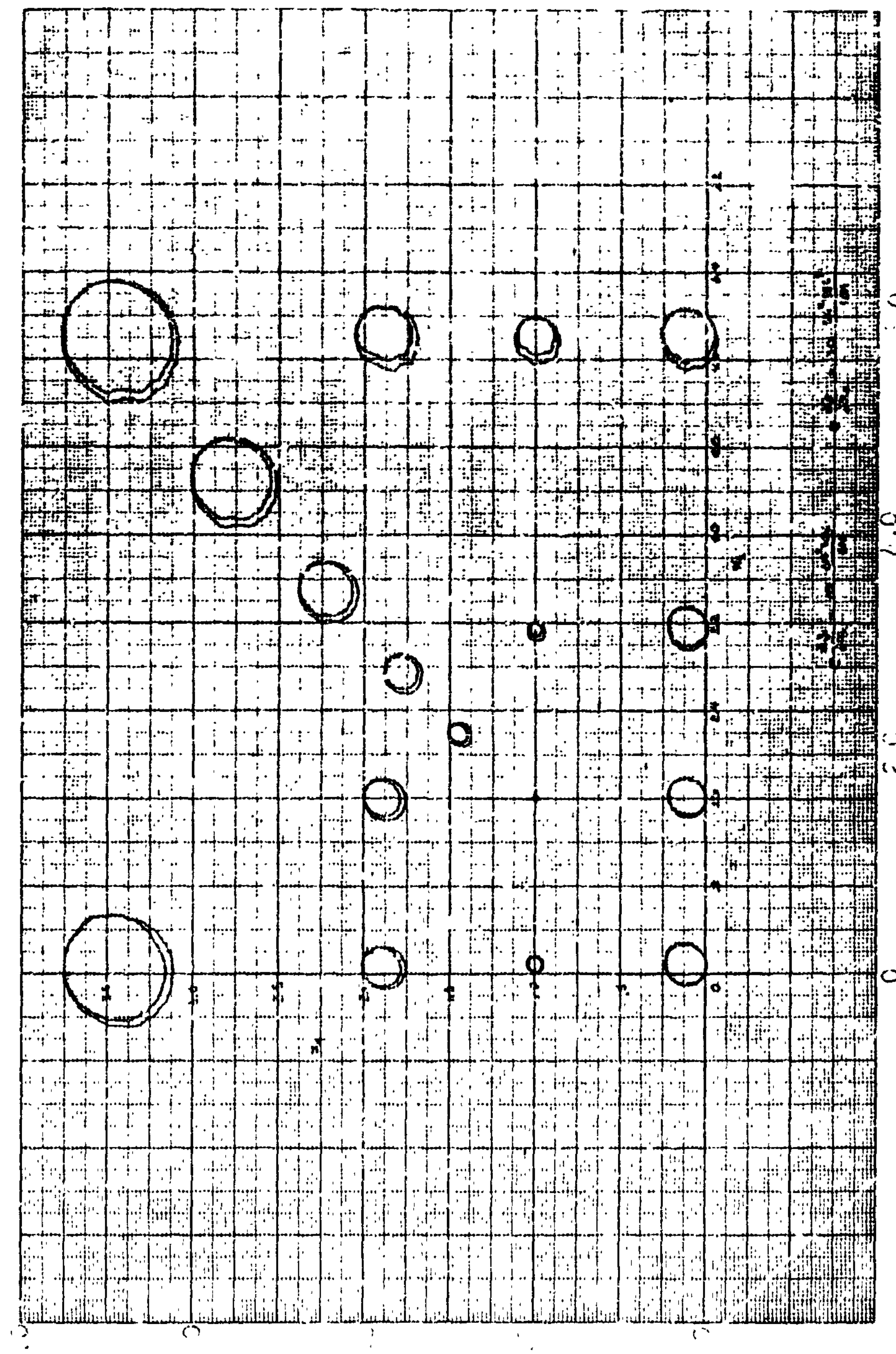
4.4 Time-variance of the Gradient

The non-uniform, seemingly erratic character (vignetting) of the descent curves (see Figures 1, 2, etc.) in the parameter space was investigated in order to clarify underlying causes and to understand the theoretical and practical problems inherent in continuous model matching.

In order to demonstrate the result of gradient computation (without introducing problems due to closed-loop adjustment of parameters) several open-loop gradient loci were plotted. Figure 28 shows these loci at various trial points in the α_3, α_4 plane. The parameters α_1, α_2 were set at their "correct" values equal to the system parameters a_1, a_2 respectively. A sinusoidal excitation signal x was used in these computations.

Figure 28 shows an interesting and unexpected result: The α_3, α_4 loci of the time-variant gradient vector $(\partial F/\partial \alpha_3, \partial F/\partial \alpha_4)$ are nearly circular Lissajous figures describing two full rotations for every period of the

Open Loop Gradient Loci in the a_3, a_4 Plane



a_3

Figure 28

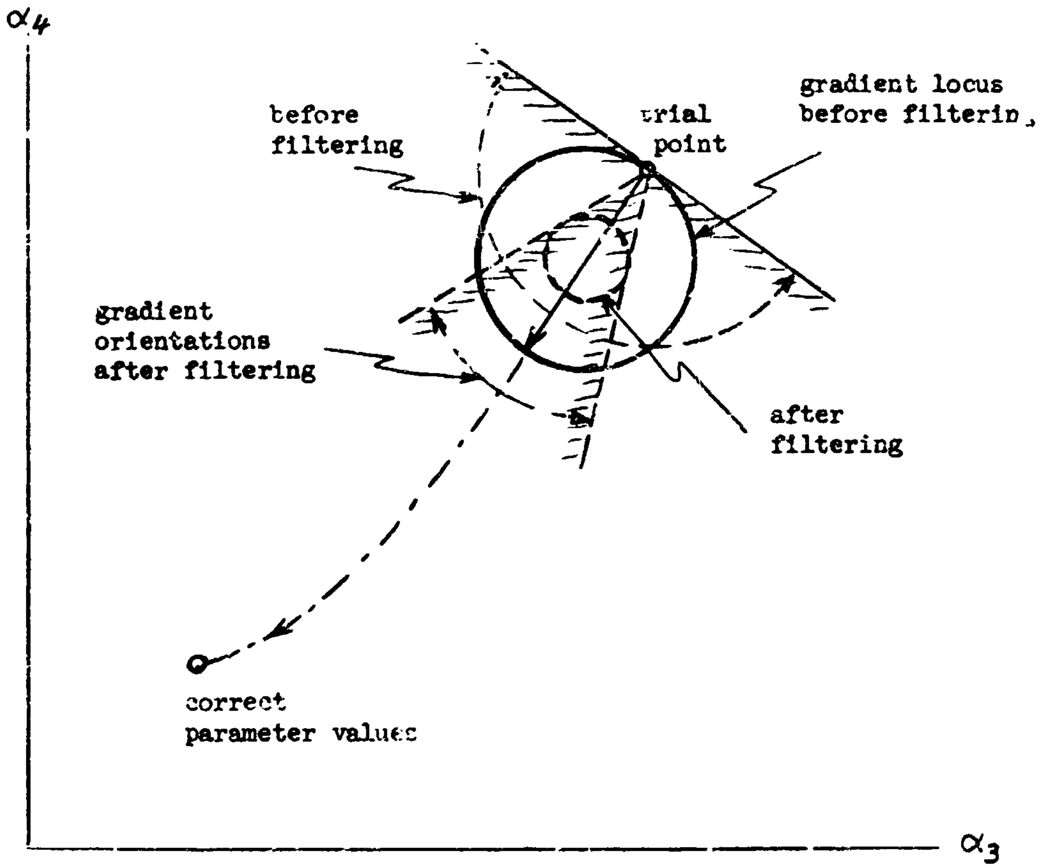
sinusoidal excitation signal. The gradient loci pass through the trial point during every full rotation. At these instances the gradient has magnitude zero, corresponding to the cusps in the vignetting descent paths (Figures 1, 2). For random excitation of the model matching system the loci are Lissajous figures of irregular shape with varying time-interval per full rotation, i.e., between passages through the trial point. The implications of this result are of great significance. It can be observed that the time-varying gradient sweeps an angular domain of 180 degrees in the α_3, α_4 plane and that only the mean orientation of the gradient vector points in the direction in which one desires the descent to proceed. In other words, the criterion function forms a time-varying surface with local ridges and depressions which are not relevant for purposes of descent to the minimum point at $\alpha_3 = a_3$ and $\alpha_4 = a_4$. This suggests the use of filtering of the time signals which generate the gradient components $\partial F/\partial \alpha_i$ in order to emphasize the preferred mean gradient orientation. Figure 29 is a sketch of open-loop gradient loci obtained with and without low pass filtering.

A mathematical explanation of the observed phenomena can be readily provided. With a pure sinusoidal excitation $x(t)$ all output variables i.e., z, u_1, u_2, ... are sinusoids of the same frequency after the initial transients have decayed, assuming a dynamically stable system. The model matching error, $\epsilon = z - y$, is sinusoidal as well. Thus the gradient component

$$\frac{\partial F}{\partial \alpha_i} = \epsilon \frac{\partial z}{\partial \alpha_i} = \epsilon u_i$$

is a sinusoid with twice the excitation frequency, changing signs four times during each period of excitation, i.e., twice for $\epsilon = 0$, and twice for $u_i = 0$. Since the factor ϵ is common to all gradient components, the gradient must be zero periodically at times when the criterion function $F = 1/2\ \epsilon^2$ also has the value zero. In the case of random excitation the above arguments remain essentially valid, except the zeros of the gradient occur at random time intervals.

In Figure 28 it is interesting to note that the gradient samples $\left(\frac{\partial F}{\partial \alpha_3}, \frac{\partial F}{\partial \alpha_4}\right)$ plotted in a large area of the α_3, α_4 plane near the minimum of F are extremely small. This demonstrates the poor model matching definition in the case under investigation here.



Open-Loop Gradient Loci in α_3, α_4 Plane before and after Filtering of Output Signals

Figure 29.

The time-variation of the local gradient leads to an interesting observation regarding the speed of convergence for a model matching problem with many parameters. The probability of the gradient being oriented within the preferred angular range is less than one inasmuch as the vector points in other directions part of the time. If, for example, it points in the desired direction only 50% of the time in a two-parameter adjustment problem, this probability is reduced further in a three-parameter problem, and still further in a four-parameter problem, since the desired direction occupies less and less of the total angle in the hyper-space over which the gradient is free to wander. As a result, the settling time increases at least in proportion with the number of parameters to be adjusted simultaneously. On theoretical grounds the time would tend to increase with powers of 2^n where n is the number of parameters, considering the geometry of angular regions in a hyper-space. Refined filtering techniques are required to counteract this tendency. Further investigation along these lines will be required to cope with the practical implications of this result.

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